Math 5102 **Homework 4**
due Fri March 6, 2015

This homework will guide you through the proof of pointwise convergence of the Fourier series of a periodic, twice differentiable function.

1. Let $f \in C[-\pi, \pi]$. Show that $f \in L^p[-\pi, \pi]$ for any $p \in [1, \infty)$.

2. Let $f, g \in C[-\pi, \pi]$. Show that if the L^p norm $\|f - g\|_p$ is zero and $p \in [1, \infty)$, then $f(x) = g(x)$ for all $x \in [-\pi, \pi]$.

3. Let f be a periodic function in $C^2[-\pi, \pi]$ (f and two derivatives are continuous and periodic). Show that there is a constant C such that $|\hat{f}_n| \leq C/n^2$ for any $n \neq 0$.

4. Let $g(x) = \sum_{-\infty}^{\infty} \hat{f}_n e^{inx}$. Show that this series converges absolutely and uniformly. Show that g is a continuous function, periodic on $[-\pi, \pi]$.

5. Show that $\hat{g}_n = \hat{f}_n$, $\forall n \in \mathbb{Z}$. Use Plancherel's theorem and Problem 2 to show that $f(x) = g(x)$ for all $x \in [-\pi, \pi]$.

Bonus of 10p for complete solutions to all five problems, with no (serious) mistakes!