

Math 5102  
due Fri **April 17**, 2015

### Homework 6

1. Find the Green's function for the Bessel operators

a)  $Lu = -\frac{d}{dx}x\frac{du}{dx}$

b)  $Lu = -\frac{d}{dx}x\frac{du}{dx} + \frac{n^2}{x}u$

with the boundary conditions  $u(0)$  finite and  $u(1) = 0$ .

2. a) Find the Green's function for the problem

$$L = -\frac{d^2}{dx^2} + \omega^2, \quad u(a) = 0, \quad u(b) = 0$$

for  $a < b$  and  $\omega$  a constant.

b) Does this Green's function exist for all values of  $\omega$ ? If not, then what are the exceptional values for  $\omega$ ?

3. Find (as an integral) the solution to the boundary value problem

$$u'' + 3u' = f, \quad u(0) = 1, \quad u'(1) = 6$$

4. Find the Green's function for the problem

$$u''' = f, \quad u(0) = 0, \quad u''(0) = 0, \quad u'(1) = 0$$