Math 5102 **Homework 7**
due Fri **April 8**, 2019

The Fourier transform of $f(x)$, in frequency convention, has the form $\hat{f}(\xi) = \int_{-\infty}^{\infty} \mathrm{e}^{-2\pi i x\xi} f(x)dx$, with the inverse $f(x) = \int_{-\infty}^{\infty} \mathrm{e}^{2\pi i x\xi} \hat{f}(\xi)d\xi$. Use this convention to compute the following.

1. Show that the inverse Fourier transform of $\dfrac{1}{a + 2\pi i\xi}$ (where $a \in \mathbb{R}$) is $e^{-ax}\Theta(x)$ (where Θ denotes the Heaviside function).

Hint. It may be easier to calculate the Fourier transform of $e^{-ax}\Theta(x)$.)

2. Show that the inverse Fourier transform of $\dfrac{2a}{a^2 + 4\pi^2\xi^2}$ (where $a \in \mathbb{R}$) is $e^{-a|x|}$.

3. Show that the Fourier transform of e^{-ax^2} (where $a > 0$) is $\dfrac{\sqrt{\pi}}{\sqrt{a}}\mathrm{e}^{-\frac{\pi^2\xi^2}{a}}$.

Hint. $\int_{-\infty}^{\infty} \mathrm{e}^{-t^2}\, dt = \sqrt{\pi}$.

4. Show that the Fourier transform of $\frac{1}{x}$, as a distribution, is $-i\pi\mathrm{sgn}(\xi)$. Explain why we need to consider distributions.

5. Show that the Fourier transform of $\delta^{(n)}(x)$ is $(2\pi i\xi)^n$.