Math 5102 **Homework 2**

due Fri Feb 13, 2019

1. a) Show that the functions

$$\sin \frac{n\pi x}{c}, \quad n = 1, 2, \ldots \tag{1}$$

are orthogonal in $L^2[0, c]$. Normalize them, to obtain an orthonormal set.

b) We will see that it is also complete, hence (1) is an orthogonal basis, and we can expand functions $f \in L^2[0, c]$ in sine-series as $f = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c}$. Find a formula for the coefficients b_n.

c) Deduce that any *odd* function in $L^2[-c, c]$ can be expanded as a sine-series.

2. a) Show that the functions

$$1, \ \cos \frac{n\pi x}{c}, \quad n = 1, 2, \ldots \tag{2}$$

are orthogonal in $L^2[0, c]$. Normalize them, to obtain an orthonormal set.

b) We will see that it is also complete, hence (2) is an orthogonal basis, and we can expand any $f \in L^2[0, c]$ in cosine-series as $f = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c}$. Find a formula for the coefficients a_n.

c) Deduce that any *even* function in $L^2[-c, c]$ can be expanded as a cosine-series.

3. Find the sine-series of $\cos \frac{k\pi x}{c}$ for $x \in [0, c]$.

4. Assuming that (1) is an orthogonal basis in $L^2[0, c]$, and that also (2) is one, deduce that any function $f \in L^2[-c, c]$ can be expanded in a Fourier series

$$f = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{c} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{c}$$