Math 5102 **Supplementary homework**

1. The function $\mathrm{Ei}(1, y)$ is defined, for $y > 0$, by

$$\mathrm{Ei}(1, y) = \int_y^\infty \frac{\mathrm{e}^{-s}}{s}\, \mathrm{d}s$$

(a) Use integration by parts to show that, for large y, $\mathrm{Ei}(1, y)$ has the asymptotic expansion

$$\mathrm{Ei}(1, y) \sim -\sum_{n=0}^{\infty} \frac{n! \exp(-y)}{(-y)^{n+1}}$$

(b) Show that the function $\mathrm{Ei}(1, y)$ always lies between two successive terms of its asymptotic expansion.
(c) What accuracy does (b) guarantee for the numerical value of $\mathrm{Ei}(1, 10)$?
(d) Change variables to $s = y(1 + p)$ and apply Watson's Lemma to obtain the results in (a),(b).

2. The incomplete Gamma function is defined, for $y > 0$, by

$$\Gamma(1 - m, y) = \int_y^\infty s^{-m} e^{-s} ds$$

Make a change of variables as in problem 1 to find explicitly all the coefficients of the asymptotic series for large y of $e^y \Gamma(1 - m, y)$.