Homework 10, the Cauchy Principal Value integral

Problems: Folland, **2,6,7 (p. 289), 13 (p. 291).**

Turn in:
(a) Consider the family $F_\epsilon(x) = \dfrac{x}{x^2+\epsilon^2}$. (Note that, as $\epsilon \to 0$, $F_\epsilon(x)$ converges to x^{-1} on $\mathbb{R}\setminus\{0\}$.) Show that $F_\epsilon(x)$ converges in $\mathcal{D}'$ to the distribution

$$L(\phi) := -\int_{-\infty}^{\infty} \phi'(s)\ln|s|ds$$

Define the order of a distribution as the minimal $N(K)$ as in Proposition 149. Clearly if $K=[a,b]$, then the order of L is at most 1. Show that the order is zero if $0 \notin [a,b]$.

(b) Prove the following formula for L,

$$L(\phi) = \lim_{\delta\to 0}\int_{|s|>\delta} \frac{\phi(s)}{s} ds$$

(This limit is called the Cauchy principal value of the integral, denoted $PV\int_{-\infty}^{\infty} s^{-1}\phi(s)ds$.)
Show that $N(K)$ is not zero for symmetric intervals, $K=[-a,a]$.