Homework 2

01/24/2018

Folland Chapter 5: 56, 57, 58, 62

Let $\mathcal{H}$ be a Hilbert space and $E = \{e_n\}_{n\in\mathbb{N}}$ an orthonormal set in $\mathcal{H}$. Check that E is a closed bounded set in $\mathcal{H}$ which is **not** compact.

Now, let $\{\epsilon_k\}_{k\in\mathbb{N}}$ be a sequence of positive numbers and define the set

$$C = \left\{ \sum_{k\in\mathbb{N}} c_k e_k \in \mathcal{H} : (\forall k) \;\; (e_k \in E \,\&\, |c_k| \le \epsilon_k) \right\}$$

Show that C is compact iff $\sum_{k\ge 0} |\epsilon_k|^2 < \infty$.

(Always try to find simple solutions; the problems are meant to allow for them.)