Homework 9, the Weierstrass approximation theorem

Problems: Folland, **20, 22 (p. 256) 25,26 , (p. 262)**

Turn in:

Define

$$L_n(x) = \begin{cases} a_n(1-x^2)^n & \text{if } x \in [-1,1] \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

where a_n is chosen so that $\int_{\mathbb{R}} L_n(s)ds = 1$. Show that $a_n \leq (n+1)/2$ (with more work, by induction, one can check that $a_n = \frac{(2n+1)(2n)!}{(n!)^2 2^{2n+1}}$, but we don't need the exact formula here.)

(a) Show that the family $\{L_n\}_{n\geq 0}$ is an approximation to the identity (as $n \to \infty$).

(b) Use (a) to show that, if f is in $C_c([-1/2, 1/2])$, then $L_n * f$ is a sequence of polynomials which converges uniformly to f on $[-1/2, 1/2]$.