Asymptotic and transasymptotic matching; formation of singularities

\[
\frac{1}{h(\xi_s)} = \left( A - \frac{109}{10} \right)^2 + \frac{1}{12x^2} + O(1/x^3)
\]

whence \( A = \frac{109}{10} \) (because \( 1/h \) is analytic at \( \xi_s \)) and we have

\[
\xi_s = 12 + \frac{109}{10x} + O(x^{-2})
\]

(6.70)

FIGURE 6.4: Poles of (4.85) for \( C = -12 \) (\( \diamond \)) and \( C = 12 \) (+), calculated via (6.70). The light circles are on the second line of poles.

Given a solution \( h \), the constant \( C \) in (6.13) for which (6.67) holds can be calculated from asymptotic information in any direction above the real line by near least term truncation, namely

\[
C = \lim_{\arg(x) = \phi} \exp(x)x^{1/2} \left( h(x) - \sum_{k \leq |x|} \frac{\tilde{h}_{0,k}}{x^k} \right)
\]

(6.71)

(this is a particular case of much more general formulas [19] where \( \sum_{k>0} \tilde{h}_{0,k}x^{-k} \) is the common asymptotic series of all solutions of (4.85) which are small in \( \mathbb{H} \).)