PEAK 2017: Schedule, Titles and abstracts

Workshop on Perspectives and Emerging Topics in Algebra and Convexity

Haus Bergkranz, Austria
February 3-9, 2017

TIME	SATURDAY	SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY
9:30-10:30	Juhnke-Kubitzke	Draisma	Free morning	Leykin	Popescu-Pampu	Free morning
11:00-12:00	Rincon	Kubjas	—	Sanyal	Kahle	—
12:00-15:00	Lunch break	Lunch break	Lunch break	Lunch break	Lunch break	Lunch break
15:00-16:00	Riener	Blekherman	Maclagan	Süß	Shaw	—
16:30 -17:30	Bayer	Manon	Nill	Postinghel	Ilten	Free afternoon
18:00-19:30	Dinner	Dinner	Dinner	Dinner	Dinner	—
20:00-21:00	5 min. intros	—	Night Session	Night Session	Party	—

The breakfast buffet is open from 8.00 - 9.00 a.m. during which time you can prepare your own lunch packet. The evening meal is served at 6 p.m. Afterwards please completely clear your table.

Note: We might adjust the schedule to take advantage of local weather conditions.

Titles and Abstracts

1. **Five minute introductions**

   What mathematical problems would you like to discuss with (at least) some of the conference participants? This very informal session will be an entertaining sequence of five minutes talks providing various answers to this question. A typical talk might introduce an open problem, present a conjecture, highlight an ongoing collaboration, or showcase a theorem. Time limits will strictly enforced. If you would like to give such an introduction, then please inform the organizers.

2. **Brill-Noether via wall-crossing** (Arend Bayer)

   There are two classical approaches to proving Brill-Noether results: degeneration to singular curves, and using vector bundles on surfaces. The former method has now been pushed much further by tropical geometry; I will explain how the latter can be naturally phrased and extended via wall-crossing in the derived of the surface. This naturally leads to reprove (and strengthen) Lazarsfeld's classical result for curves on $K3$ surfaces, and to new results on abelian surfaces. The latter is joint work with Chunyi Li.

3. **Do Sums of Squares Dream of Free Resolutions?** (Greg Blekherman)

   I will explain a new connection between convexity properties of the cone of sums of squares on a real variety (reduced scheme) and homological properties of its free resolution, such as regularity and property $N_{2,p}$. I will discuss a specific example of square-free monomial ideals, which connects to positive semidefinite matrix completion problems. Finally I will present some open problems. Based on joint work with Rainer Sinn and Mauricio Velasco.

4. **Frobenius flocks** (Jan Draisma)

   In characteristic zero, every algebraic matroid admits a linear representation. In positive characteristic, it turns out that every algebraic matroid admits a representation by a /Frobenius flock/: a lattice worth of vector spaces that are connected by two simple axioms. These two axioms lead to a surprisingly rich theory: flocks always define matroids, they have contractions, deletions, and dual flocks, and they give rise to a partition of the lattice into cells that are both max-plus and min-plus closed. Flocks arise from other sources than algebraic matroids, as well, e.g. from linear spaces over valued fields—and even the Vamos matroid is flock-representable! Certain matroids are so rigid, that the cell structure of any flock representing them is necessarily a fan, and in this case the matroid is algebraic if and only if it admits a linear representation. This leads to new results of non-algebraicity for a large class of matroids.

5. **Product Ranks and Linear Subspaces of Special Hypersurfaces** (Nathan Ilten)

   The product or Chow rank of a homogeneous polynomial f is the smallest number of summands needed to express it as a sum of products of linear forms. In this talk, I will discuss recent work of myself with Hendrik Süß and Zach Teitler providing non-trivial lower bounds for product rank. The key idea is to compare linear subspaces contained in the hypersurface defined by f with linear subspaces contained in the vanishing locus of a generic product of linear forms. It turns out that many of the linear subspaces contained in the vanishing locus of a generic product of linear forms must have a very special form. This leads to new lower bounds for the product ranks of the 3×3 determinant and permanent, the 4×4 permanent, and the 6×6 Pfaffian.

6. **Face numbers of (balanced) manifolds** (Martina Juhnke-Kubitzke)

   The Dehn-Sommerville relations are one of the most famous symmetries in the theory of face enumeration. They were first proved by Dehn and Sommerville for simplicial polytopes and later generalized to all triangulated manifolds. Recently, Murai, Novik and Yoshida found a simple algebraic way to express the Dehn-Sommerville equation for triangulated manifolds by using Matlis duality and Stanley–Reisner theory. I will shortly describe their results and then give applications to face numbers of balanced manifolds (with and without boundary). This is joint work with Satoshi Murai, Isabella Novik and Connor Sawaske.

7. **Hyperbolic Coxeter groups and commutative algebra** (Thomas Kahle)

   We show that the virtual cohomological dimension of a right-angled Coxeter group equals the maximal regularity of the Stanley–Reisner ring of its nerve (the maximum is taken over coefficient fields). This gives a bridge to transport results from geometric group theory to commutative algebra and vice versa. For example, we can show that for any integers p, r there exists monomial ideals generated in degree two and with a resolution that is linear for p steps, but has regularity r. This is joint work with Alexandru Constantinescu and Matteo Varbaro.

8. **How to flatten a soccer ball** (Kaie Kubjas)

   In this talk, I will explain how to compute the image of a simple semialgebraic set of 3-space ("soccer ball") under a polynomial map into the plane. In general cases, the boundary of the image is given by two highly singular curves. I will discuss the number of connected components of the complement of the image, maps onto convex polygons as well as connections to convex optimization. This talk is based on joint work with Pablo A. Parrilo and Bernd Sturmfels.

9. **Beyond Polyhedral Homotopies** (Anton Leykin)

   Given an affine variety we consider the problem of finding isolated solutions, on the given variety, of a polynomial system that is sparse with respect to a non-monomial basis. We construct a method to solve this problem using tropical geometry and homotopy continuation machinery. As a part of our algorithm we also get the mixed volume of Newton–Okounkov bodies without having to compute the bodies. This approach generalizes the polyhedral homotopies by Huber and Sturmfels. This is joint work with Josephine Yu.

10. **Tree compactifications of the moduli space of genus zero curves** (Diane Maclagan)

    The moduli space $M_{0,n}$ of smooth genus zero curves with n marked points has a standard compactification by the Deligne-Mumford module space of stable genus zero curves with n marked points. The compactification can be constructed as the closure of $M_{0,n}$ inside a toric variety. The fan of the toric variety is moduli space of phylogenetic trees. I will discuss joint work with Dustin Cartwright to construct other compactifications of $M_{0,n}$ by varying the toric variety by using variants of phylogenetic trees. These compactifications include many of the standard alternative compactifications of $M_{0,n}$.

11. **Khovanskii bases, higher rank valuations and tropical geometry** (Chris Manon)

    The notion of Khovanskii basis is a generalization SAGBI bases to algebras with a choice of discrete valuation, usually taken to have maximal rank. In a very general setting, an affine variety possesses a flat degeneration to a toric variety provided its coordinate ring contains a finite Khovanskii basis. This in turn enables the use of numerous computational and combinatorial techniques. I'll describe a necessary and sufficient condition for the existence of a finite Khovanskii basis framed in the language of tropical geometry. This is joint work with Kiumars Kaveh.

12. **Ehrhart theory of spanning lattice polytopes** (Benjamin Nill)

    We call a lattice polytope spanning if its lattice points affinely generate the lattice. In joint work with Johannes Hofscheier and Lukas Katthän, we show that the Ehrhart $h*$-vector of a spanning lattice polytope has no gaps. I will discuss background and motivation of this result and the methods of its proof. In particular, I will explain how this generalizes a result by Blekherman, Smith, Velasco and why it implies a polyhedral version of the Eisenbud-Goto conjecture.

13. **Variations on inversion theorems for Newton–Puiseux series** (Patrick Popescu-Pampu)

    Let $f(x, y)$ be an irreducible formal power series without constant term, over an algebraically closed field of characteristic zero. One may solve the equation $f(x, y) = 0$ by choosing either x or y as independent variable, getting two finite sets of Newton–Puiseux series. In 1967 and 1968 respectively, Abhyankar and Zariski published proofs of an inversion theorem, expressing the characteristic exponents of one set of series in terms of those of the other set. In fact, a more general theorem, stated by Halphen in 1876 and proved by Stolz in 1879, relates also the coefficients of the characteristic terms of both sets of series. This theorem seems to have been completely forgotten. I will sketch two new proofs of it and I will explain how it may be generalized into a theorem concerning irreducible series with an arbitrary number of variables. This is joint work with Evelia García Barroso and Pedro González Pérez.

14. **Tropical compactifications, Mori Dream Spaces and Minkowski bases** (Elisa Postinghel)

    Given a Mori Dream Space X, we construct via tropicalisation a model dominating all the small $\mathbb{Q}$-factorial modifications of X. Via this construction we recover a Minkowski basis for the Newton-Okounkov bodies of Cartier divisors on X and hence generators of the movable cone of X. This is joint work with Stefano Urbinati.

15. **Quadrature on curves and cubature on the plane via conic optimization and real algebraic geometry** (Cordian Riener)

    Let d and k be positive integers. Let μ be a positive Borel measure on R^2 possessing finite moments up to degree $2d - 1$. If the support of μ is contained in an algebraic curve of degree k, then we show that there exists a quadrature rule for with at most dk many nodes all placed on the curve (and positive weights) that is exact on all polynomials of degree at most

$2d-1$. This generalizes Gaußquadrature where the curve is a line and (the odd case of) Szegö quadrature where the curve is a circle to arbitrary plane algebraic curves. We use this result to show that, without any hypothesis on the support of μ, there is always a cubature rule for with at most $3/2d(d-1)+1$ many nodes. In both results, we show that the quadrature or cubature rule can be chosen such that its value on a fixed positive definite form of degree $2d$ is minimized. Our proof uses both results from convex optimisation and from real algebraic geometry. This is joint work with Markus Schweighofer.

16. **Tropical Ideals** (Felipe Rincon)

    We study a special class of ideals, called tropical ideals, in the semiring of tropical polynomials, with the goal of developing a useful and solid algebraic foundation for tropical geometry. The class of tropical ideals strictly includes the tropicalizations of classical ideals, and it satisfies many desirable properties that mimic the classical setup. In particular, every tropical ideal has an associated variety, which we prove is always a finite polyhedral complex. In addition we show that tropical ideals satisfy the ascending chain condition, even though they are typically not finitely generated, and also the weak Nullstellensatz. This is joint work with Diane Maclagan.

17. **Combinatorially mixed valuations on polytopes** (Raman Sanyal)

    The confluence of Minkowski addition and volume gives rise to the vast theory of mixed volumes. A fundamental result is that mixed volumes are linear, nonnegative, and monotone. In the discrete setting (i.e. lattice polytopes), discrete volume (i.e counting lattice points) takes the place of the volume. The resulting discrete mixed volumes are linear by construction but nonnegativity and monotonicity is genuinely lost. In this talk I will advertise the notion of combinatorially mixed valuations associated to valuations on (lattice) polytopes. For general polytopes, the theory of combinatorially mixed valuations parallels that of mixed valuations. For lattice polytopes, many combinatorially mixed valuations (including the combinatorial mixed volume) are nonnegative and monotone and the study of general combinatorially mixed valuations is strongly tied to the combinatorics of subdivisions of lattice polytopes. This is joint work with Katharina Jochemko.

18. **Properties of tropical cohomology** (Kristin Shaw)

    Tropical cohomology groups, as introduced by Itenberg, Katzarkov, Mikhalkin, and Zharkov, give a way of obtain Hodge theoretic invariants from polyhedral geometry.

    In this talk I will explain some different approaches to this cohomology theory, via singular and cellular cohomology and also superforms. I will discuss some properties of these groups and focus on open questions about them and their variants.

19. **Semi-toric integrable systems and toric degenerations** (Hendrik Süß)

    Smooth toric varieties come naturally with the structure of an integrable system by considering the moment map of the torus action. In the case of lower dimensional torus actions the situation is more complicated. Here, a recent result by Hohloch, Sabatini, Symington and Sepe gives an exact criterion for the existence of so-called semi-toric systems on an algebraic surface with C-action. We compare symplectic and algebraic results for such surfaces and conclude that the existence of semi-toric systems on C-surfaces is equivalent to the existence of an equivariant degeneration to a normal toric variety. This is joint work with Christophe Wacheux.