Maria Angelica Cueto

List of Publications with Abstracts

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Preprints:

1. **How to repair tropicalizations of plane curves using modifications** (with H. Markwig). Accepted for publication at *Experimental Mathematics* (2015). Eprint: `arXiv:1409.7430`

   Tropical geometry is sensitive to embeddings of algebraic varieties inside toric varieties. The purpose of this paper is to advertise tropical modifications as a tool to locally repair bad embeddings of plane curves, allowing the re-embedded tropical curve to better reflect the geometry of the input curve. Our approach is based on the close connection between analytic curves (in the sense of Berkovich) and tropical curves. We investigate the effect of these tropical modifications on the tropicalization map defined on the analytification of the given curve.

   Our study is motivated by the case of plane elliptic cubics, where good embeddings are characterized in terms of the j-invariant. Given a plane elliptic cubic whose tropicalization contains a cycle, we present an effective algorithm, based on non-Archimedean methods, to linearly re-embed the curve in dimension 4 so that its tropicalization reflects the j-invariant. We give an alternative elementary proof of this result by interpreting the initial terms of the A-discriminant of the defining equation as a local discriminant in the Newton subdivision.

2. **Cremona linearizations of some classical varieties** (with Ciro Ciliberto, Massimiliano Mella, Kristian Ranestad and Piotr Zwiernik). Accepted for publication at the *Proceedings of the Conference "Homage to Corrado Segre"* (Torino, Italy), edited by Birkäuser (2014). Eprint: `arXiv:1403.1814`.

   In this paper we present an effective method for linearizing rational varieties of codimension at least two under Cremona transformations, starting from a given parametrization. Using these linearizing Cremonas, we simplify the equations of secant and tangential varieties of some classical examples, including Veronese, Segre and Grassmann varieties. We end the paper by treating the special case of the Segre embedding of the n-fold product of projective spaces, where cumulant Cremonas, arising from algebraic statistics, appear as specific cases of our general construction.

3. **Implicitization of surfaces via geometric tropicalization**. Submitted (2012).
   Eprint: `arXiv:1105.0509`.

   In this paper we further develop the theory of geometric tropicalization due to Hacking, Keel and Tevelev and we describe tropical methods for implicitization of surfaces. More precisely, we enrich this theory with a combinatorial formula for tropical multiplicities of regular points in arbitrary dimension and we prove a conjecture of Sturmfels and Tevelev regarding sufficient combinatorial conditions to compute tropical varieties via geometric tropicalization. Using these two results, we extend previous work of Sturmfels, Tevelev and Yu for tropical implicitization of generic surfaces, and we provide methods for approaching the non-generic cases.

Books:

1. **Algebraic and Combinatorial Aspects of Tropical Geometry** (Proceedings of the CIEM Workshop in tropical geometry), (E. Brugallé, M.A. Cueto, A. Dickenstein, E.-M. Feichtner and I. Itenberg, eds.), AMS, *Contemporary Mathematics* **589** (2013), x+350 pp, `doi:10.1090/conm/589`.

This volume contains the proceedings of the CIEM workshop on Tropical Geometry, held December 1216, 2011, at the International Centre for Mathematical Meetings (CIEM), Castro Urdiales, Spain.

Tropical geometry is a new and rapidly developing field of mathematics which has deep connections with various areas of mathematics and physics, such as algebraic geometry, symplectic geometry, complex analysis, dynamical systems, combinatorics, statistical physics, and string theory. As reflected by the content of this volume, this meeting was mainly focused on the geometric side of the tropical world with an emphasis on relations between tropical geometry, algebraic geometry, and combinatorics.

This volume provides an overview of current trends concerning algebraic and combinatorial aspects of tropical geometry through eleven papers combining expository parts and development of modern techniques and tools.

Research Articles:

1. **Faithful tropicalization of the Grassmannian of planes** (with Mathias Häbich and Annette Werner). *Mathematische Annalen* **360**(1–2), 391–437, `doi 10.1007/s00208-014-1037-3`.

   We show that the tropical projective Grassmannian of planes is homeomorphic to a closed subset of the analytic Grassmannian in Berkovich's sense by constructing a continuous section to the tropicalization map. Our main tool is an explicit description of the algebraic coordinate rings of the toric strata of the Grassmannian. We determine the fibers of the tropicalization map and compute the initial degenerations of all the toric strata. As a consequence, we prove that the tropical multiplicities of all points in the tropical projective Grassmannian are equal to one. Finally, we determine a piecewise linear structure on the image of our section that corresponds to the polyhedral structure on the tropical projective Grassmannian.

2. **Mixed discriminants** (with Eduardo Cattani, Alicia Dickenstein, Sandra Di Rocco and Bernd Sturmfels). *Mathematische Zeitschrift* **274**(3) (2013), 761–778. `doi:10.1007/s00209-012-1095-8`.

   The mixed discriminant of n Laurent polynomials in n variables is the irreducible polynomial in the coefficients which vanishes whenever two of the roots coincide. The Cayley trick expresses the mixed discriminant as an A-discriminant. We show that the degree of the mixed discriminant is a piecewise linear function in the Plücker coordinates of a mixed Grassmannian. An explicit degree formula is given for the case of plane curves.

3. **Tropical secant graphs of monomial curves** (with Shaowei Lin). *Beiträge zur Algebra und Geometrie* **54**(1) (2013), 383–418, `doi 10.1007/s13366-012-0091-9`.
   Extended abstract published in *Proceedings 22nd International Conference on Formal Power Series and Algebraic Combinatorics (FPSAC 2010)*, *Discrete Math. Theor. Comput. Sci. Proc.*, AN, Nancy (2010), 669–680. `dmAN0147/3170`.

   The first secant variety of a projective monomial curve is a threefold with an action by a one-dimensional torus. Its tropicalization is a three-dimensional fan with one-dimensional lineality space, so the tropical threefold is represented by a balanced graph. Our main result is an explicit construction of that graph. As a consequence we obtain algorithms to effectively compute the multidegree and Chow polytope of an arbitrary projective monomial curve. This generalizes an earlier degree formula due to Ranestad. The combinatorics underlying our construction is rather delicate, and it is based on a refinement of the theory of geometric tropicalization due to Hacking, Keel and Tevelev.

4. **Tropical mixtures of star tree metrics**. *Annals of Combinatorics* **16**(2) (2012), 233–251, `doi:10.1007/s00026-012-0128-7`.

   We study tree metrics that can be realized as a mixture of two star tree metrics. We prove that the only trees admitting such a decomposition are the ones coming from a tree with at most one internal edge, and whose weight satisfies certain linear inequalities. We also characterize the fibers of the corresponding mixture map. In addition, we discuss the general framework of tropical secant varieties and we interpret our results within this setting. Finally, we show that the set of tree metric ranks of metrics on n taxa is unbounded.

5. **The maximum independent sets of De Bruijn graphs of diameter 3** (with Dustin A. Cartwright and Enrique A. Tobis). *Electronic Journal of Combinatorics* **18** (1) (2011), `paper 194`, 1–18.

   The nodes of the de Bruijn graph $B(d,3)$ consist of all strings of length 3, taken from an alphabet of size d, with edges between words which are distinct substrings of a word of length 4. We give an inductive characterization of the maximum independent sets of the de Bruijn graphs $B(d,3)$ and for the de Bruijn graph of diameter three with loops removed, for arbitrary alphabet size. We derive a recurrence relation and an exponential generating function for their number. This recurrence allows us to construct exponentially many comma-free codes of length 3 with maximal cardinality.

6. **Polyhedral geometry of phylogenetic rogue taxa** (with Frederick Matsen). *Bulletin of Mathematical Biology* **73**(6) (2010), 1202–1226. `doi:10.1007/s11538-010-9556-x`.

   It is well known among phylogeneticists that adding an extra taxon (e.g. species) to a data set can alter the structure of the optimal phylogenetic tree in surprising ways. However, little is known about this "rogue taxon" effect. In this paper we characterize the behavior of the balanced minimum evolution (BME) phylogenetics on data sets of this type using tools from polyhedral geometry. First we show that for any distance matrix there exist distances to a "rogue taxon" such that the BME-optimal tree for the data set with the new taxon does not contain any nontrivial splits (bipartitions) of the optimal tree for the original data. Second, we prove a theorem which restricts the topology of BME-optimal trees for data sets of this type, thus showing that a rogue taxon cannot have an arbitrary effect on the optimal tree. Third, we construct polyhedral cones computationally which give complete answers for BME rogue taxon behavior when our original data fits a tree t on four, five, and six taxa. We use these cones to derive sufficient conditions for rogue taxon behavior for four taxa, and to understand the frequency of the rogue taxon effect via simulation.

7. **An implicitization challenge for binary factor analysis** (with Enrique A. Tobis and Josephine Yu), *Journal of Symbolic Computation* **45**(12) (2010), 1296–1315.
   `doi:10.1016/j.jsc.2010.06.011`.

   We use tropical geometry to compute the multidegree of the hypersurface of a statistical model with two hidden and four observed binary random variables, solving an open question stated by Drton, Sturmfels and Sullivant in "Lectures on Algebraic Statistics"(Problem 7.7). The model is obtained from the undirected graphical model of the complete bipartite graph $K_{2,4}$ by marginalizing two of the six binary random variables. We present algorithms for computing the Newton polytope of its defining equation by parallel walks along the polytope and its normal fan. In this way we compute vertices of the polytope. Finally, we also compute and certify its facets by studying tangent cones of the polytope at the symmetry class of vertices. The Newton polytope has 17 214 912 vertices in 44 938 symmetry classes and 70 646 facets in 246 symmetry classes.

8. **Geometry of the restricted Boltzmann machine** (with Jason Morton and Bernd Sturmfels). *Algebraic Methods in Statistics and Probability II*, M. Viana and H. Wynn (eds.), American Mathematical Society, Contemporary Mathematics **516** (2010), 135–153.
   `doi:10.1090/conm/516/10172`.

   The restricted Boltzmann machine is a graphical model for binary random variables. Based on a complete bipartite graph separating hidden and observed variables, it is the binary analog to the factor analysis model. We study this graphical model from the perspectives of algebraic statistics and tropical geometry, starting with the observation that its Zariski closure is a Hadamard power of the first secant variety of the Segre variety of projective lines. We derive a dimension formula for the tropicalized model, and we use it to show that the restricted Boltzmann machine is identifiable in many cases.

9. **Some results on inhomogeneous discriminants** (with Alicia Dickenstein). *Proceedings of the XVI Coloquio Latinoamericano de Algebra (Colonia del Sacramento, Uruguay, August 2005)*, W. Ferrer Santos, G. González Sprinberg, A. Rittatore and A. Solotar (eds.), Biblioteca de la Revista Matemática Iberoamericana (2007), 42–61. Eprint: `arXiv:math.AG/0610031`.

   We study generalized Horn-Kapranov rational parametrizations of inhomogeneous sparse discriminants from both a theoretical and an algorithmic perspective. We show that all these parametrizations are birational and prove some results on the corresponding implicit equations. We also propose a combinatorial algorithm to compute the degree of inhomogeneous discriminantal surfaces associated to uniform matrices.