Alexander Petrov

Here K always means a complete discretely valued field of characteristic 0 with residue field k which is perfect of characteristic p.

Talk 1. Étale fundamental group and local systems. (Gleb Terentiuk, Michigan) Define étale fundamental group of a scheme equipped with a geometric base point, state the comparison with the topological fundamental group for varieties over $\mathbb{C}$. State the fundamental exact sequence. Compute $\pi_1^{\mathrm{et}}(\mathbb{G}_{m,\bar{F}})$ for a field F of characteristic zero, and compute the Galois action on it. Define the notion of a $\mathbb{Q}_\ell$-local system on a scheme. State that a smooth proper morphism $f : X \to S$ (where ℓ is invertible on S) gives rise to étale local systems $R^i f_* \mathbb{Q}_\ell$.

Prove that for a local system on $\operatorname{Spec} F((t))$ for a field F finitely generated over $\mathbb{Q}$ the action of the geometric fundamental group is quasi-unipotent. Deduce Grothendieck's quasi-unipotent monodromy theorem for a smooth proper morphism $X \to S$ to an open curve S over $\mathbb{C}$.

References: [Gro63] [Mil] [ST68, Appendix]; some nice notes by Jacob Tsimerman: `https://www.math.utoronto.ca/~jacobt/Lecture6.pdf`; the introduction to Weil II: `http://www.numdam.org/item/PMIHES_1980__52__137_0/`

Talk 2. p-adic Hodge theory. (Alice Lin, Harvard) Define $\mathbb{C}_p$, mention that it is algebraically closed. Define B^+_{dR} and B_{dR}, the filtrations and the Galois actions on them. State de Rham and Hodge-Tate comparison theorems for smooth proper varieties over a p-adic field. State Tate's computation of Galois invariants on $\mathbb{C}_p(i)$. For an abelian variety A over K with good reduction over $\mathcal{O}_K$ sketch the construction of the Galois-equivariant map $H^1(A, \mathcal{O}_A) \to H^1_{\mathrm{et}}(A_{\overline{K}}, \mathbb{Z}_p) \otimes_{\mathbb{Z}_p} \mathbb{C}_p$.

References: [Fon94, §1] [BC, §2, 4] [Bha, 2.2] [Tat67]

Talk 3. Rigid-analytic geometry. (Zeyu Liu, UCSD) Serre's observation that the naive definition of p-adic manifolds gives just $p-1$ isomorphism classes of n-dimensional manifolds. Define Tate algebras $\mathbb{Q}_p\langle x_1, \ldots, x_n\rangle$, $\mathbb{C}_p\langle x_1, \ldots, x_n\rangle$, and $\mathbb{C}_p\langle x, x^{-1}\rangle$, explain that elements of each of these algebras give rise to functions on sets $\{x_1, \ldots, x_n \in \mathbb{C}_p || x_i|_p \leq 1\}$ and $\{x \in \mathbb{C}_p || x|_p = 1\}$, respectively. Define affinoid algebras over K.

State that there exists the category of rigid-analytic spaces over K (you do not need to define it completely), having the following features: it admits a full subcategory of affinoid spaces, equivalent to the category opposite to the category of affionoid algebras; there is the analytification functor from algebraic varieties over K, and the Raynaud generic fiber functor from admissible formal schemes over $\operatorname{Spf} \mathcal{O}_K$.

Use Kummer theory to show that the algebra $\mathbb{C}_p\langle x\rangle$ of functions on the unit disk has many non-split finite étale extensions.

References: [Ser65] [Con] [Bos14] [dJvdP96, §4]

Talk 4: More on variations of Hodge structures, period maps. (Andy Jiang, Michigan)

State the theorem of the fixed part. Deduce that for an irreducible $\mathbb{C}$-local system on a smooth complex variety there is at most one way to equip it with a complex variation of Hodge structures, up to a shift. Prove that for an irreducible local $\mathbb{C}$-system equipped with a polarizable $\mathbb{C}$-VHS it's impossible to have gaps in Hodge weights, that is to have $H^{p+1,q-1} \neq 0, H^{p-1,q+1} \neq 0$ but $H^{p,q} = 0$. Define the period domain for a given type of Hodge structures, and the period map associated to a $\mathbb{Z}$-variation of Hodge structures on a smooth complex variety. Rephrase Griffiths transversality as a statement about the differential of the period map. Statement of Deligne's finiteness theorem for local systems underlying a $\mathbb{Z}$-VHS.

References: Chapters 4, 5, 13 of `https://www-fourier.ujf-grenoble.fr/~peters/Books/PeriodBook.f/SecondEdition/PerBook.pdf` Section 1 of Deligne's 'Un théorème de finitude pour la monodromie' `https://publications.ias.edu/sites/default/files/56_Untheoremede.pdf`

Daniel Litt

Talk 1: The classical Riemann-Hilbert correspondence. (Christian Klevdal, UCSD)

-Flat bundles and local systems

-Why are flat bundles the same as ODEs?

-The Riemann-Hilbert correspondence in the smooth proper case

-Regular singularities

–A couple examples, e.g. the flat bundle $(\mathcal{O}_{\mathbb{P}^1}, d - a \cdot dz/z)$ with regular singularities at $0, \infty$ and its monodromy, and maybe a slightly more complicated example.

-All flat bundles on smooth complex varieties are algebraizable (Deligne)

References:

- Brian Conrad's notes: `https://math.stanford.edu/~conrad/papers/rhtalk.pdf`

- Deligne's paper: `https://publications.ias.edu/node/355`, English translation: `https://labs.thosgood.com/translations/978-3-540-05190-9.pdf`

Talk 2: Variations of Hodge structure (and Higgs bundles). (Yilong Zhang, Purdue)

-Variations of Hodge structure, complex variations of Hodge structure (definition)

-If $f\colon X \to S$ is smooth and proper, $R^i f_* \mathbb{Z}$ carries a VHS

-The Gauss-Manin connection (analytic construction), Griffiths transversality

-The associated graded Higgs bundle

-What is a Higgs bundle more generally?

-Time permitting: Some examples, the non-abelian Hodge correspondence in rank one

References:

(i) Some old notes I wrote: `https://virtualmath1.stanford.edu/~conrad/shimsem/2013Notes/Littvhs.pdf`

(ii) Simpson's paper on the Ubiquity of VHS: `https://math.mit.edu/juvitop/old/notes/pspum053-1141208.pdf`

Talk 3. Rigid local systems. (Jake Huryn, Ohio State) Define the notion of a rigid local system on a variety over $\mathbb{C}$, and mention cohomological rigidity. State Simpson's motivicty conjecture. Explain that a rigid local system extends to an étale local system on the descent of the variety to a finitely generated field. Using Margulis superrigidity, describe the example of the local system of 1st cohomology of the universal abelian scheme on the moduli space $A_{g,n}$ of principally polarized abelian varieties of dimension $g > 1$.

References: [Sim92] [Mar91, Theorem IX.6.15 (ii)]

Talk 4: Algebraic differential equations in characteristic $p > 0$. (Ziquan Yang, Wisconsin)

-Flat bundles in positive characteristic (example: the canonical connection on the Frobenius pullback)

-Algebraic de Rham cohomology, algebraic nature of the Gauss-Manin connection (as a natural example of a flat bundle in positive characteristic)

-The statement of the main theorem of Deligne-Illusie (i.e. decomposability of the de Rham complex)

-The conjugate filtration and its associated graded pieces

References:

(i) Katz, nilpotent connections: `http://www.numdam.org/article/PMIHES_1970__39__175_0.pdf`

(ii) Deligne-Illusie: `https://eudml.org/doc/143480`

(iii) Katz, algebraic solutions to differential equations: `https://link.springer.com/article/10.1007/BF01389714`

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[Bha] Bhargav Bhatt. The Hodge-Tate decomposition via perfectoid space, Arizona Winter School notes. `https://swc-math.github.io/aws/2017/2017BhattNotes.pdf`.

[Bos14] Siegfried Bosch. *Lectures on formal and rigid geometry*, volume 2105 of *Lecture Notes in Mathematics*. Springer, Cham, 2014.

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[dJvdP96] Johan de Jong and Marius van der Put. Étale cohomology of rigid analytic spaces. *Doc. Math.*, 1:No. 01, 1–56, 1996.

[Fon94] Jean-Marc Fontaine. Le corps des périodes p-adiques. Number 223, pages 59–111. 1994. With an appendix by Pierre Colmez, Périodes p-adiques (Bures-sur-Yvette, 1988).

[Gro63] Alexander Grothendieck. *Revêtements étales et groupe fondamental. Fasc. I: Exposés 1 à 5*. Institut des Hautes Études Scientifiques, Paris, 1963. Troisième édition, corrigée, Séminaire de Géométrie Algébrique, 1960/61.

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[Mil] James Milne. Lecture notes on étale cohomology. https://www.jmilne.org/math/CourseNotes/LEC.pdf.

[Ser65] Jean-Pierre Serre. Classification des variétés analytiques p-adiques compactes. *Topology*, 3:409–412, 1965.

[Sim92] Carlos T. Simpson. Higgs bundles and local systems. *Inst. Hautes Études Sci. Publ. Math.*, (75):5–95, 1992.

[ST68] Jean-Pierre Serre and John Tate. Good reduction of abelian varieties. *Ann. of Math. (2)*, 88:492–517, 1968.

[Tat67] J. T. Tate. p-divisible groups. In *Proc. Conf. Local Fields (Driebergen, 1966)*, pages 158–183. Springer, Berlin-New York, 1967.