The journey of a tropical geometer through four countries

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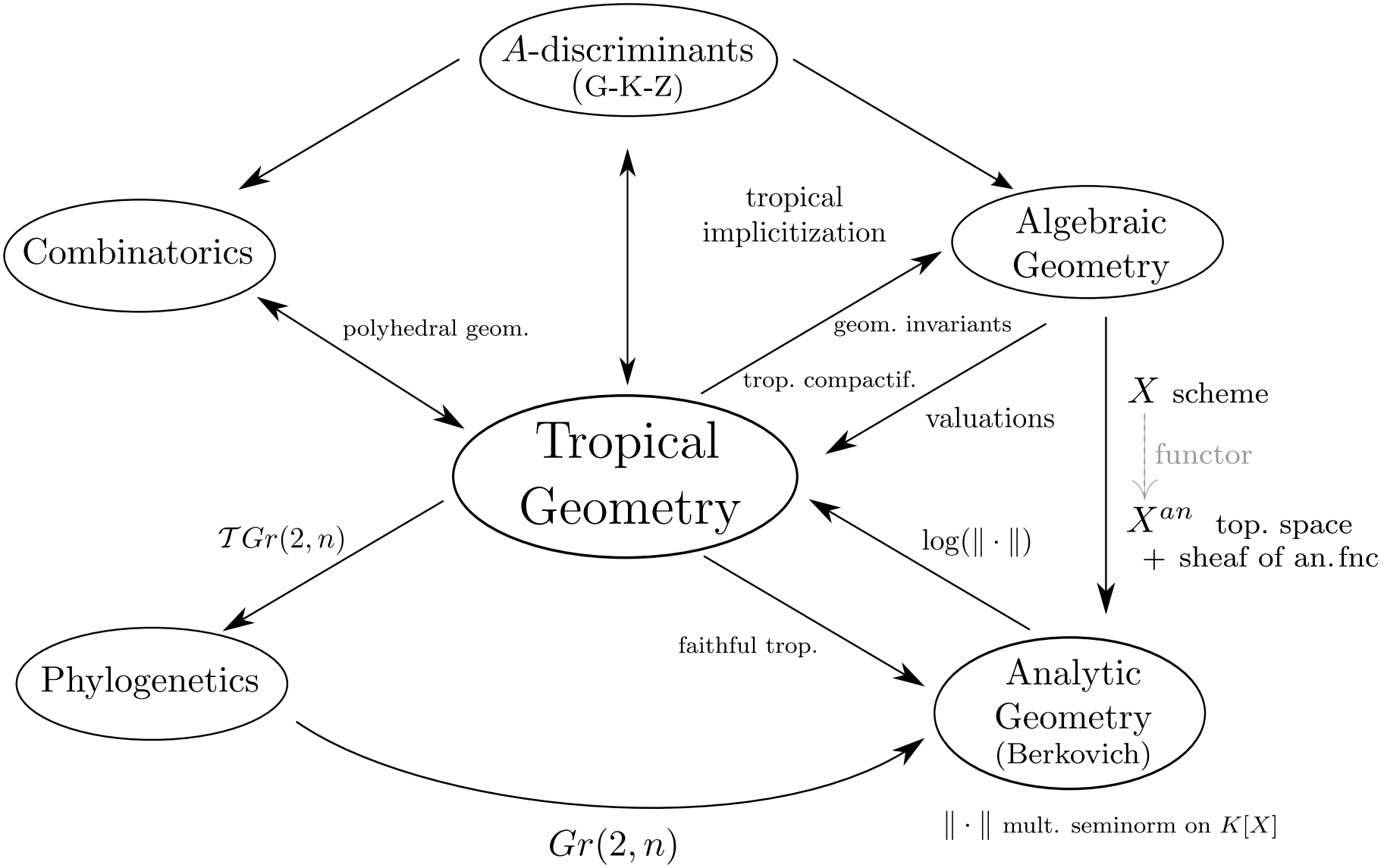
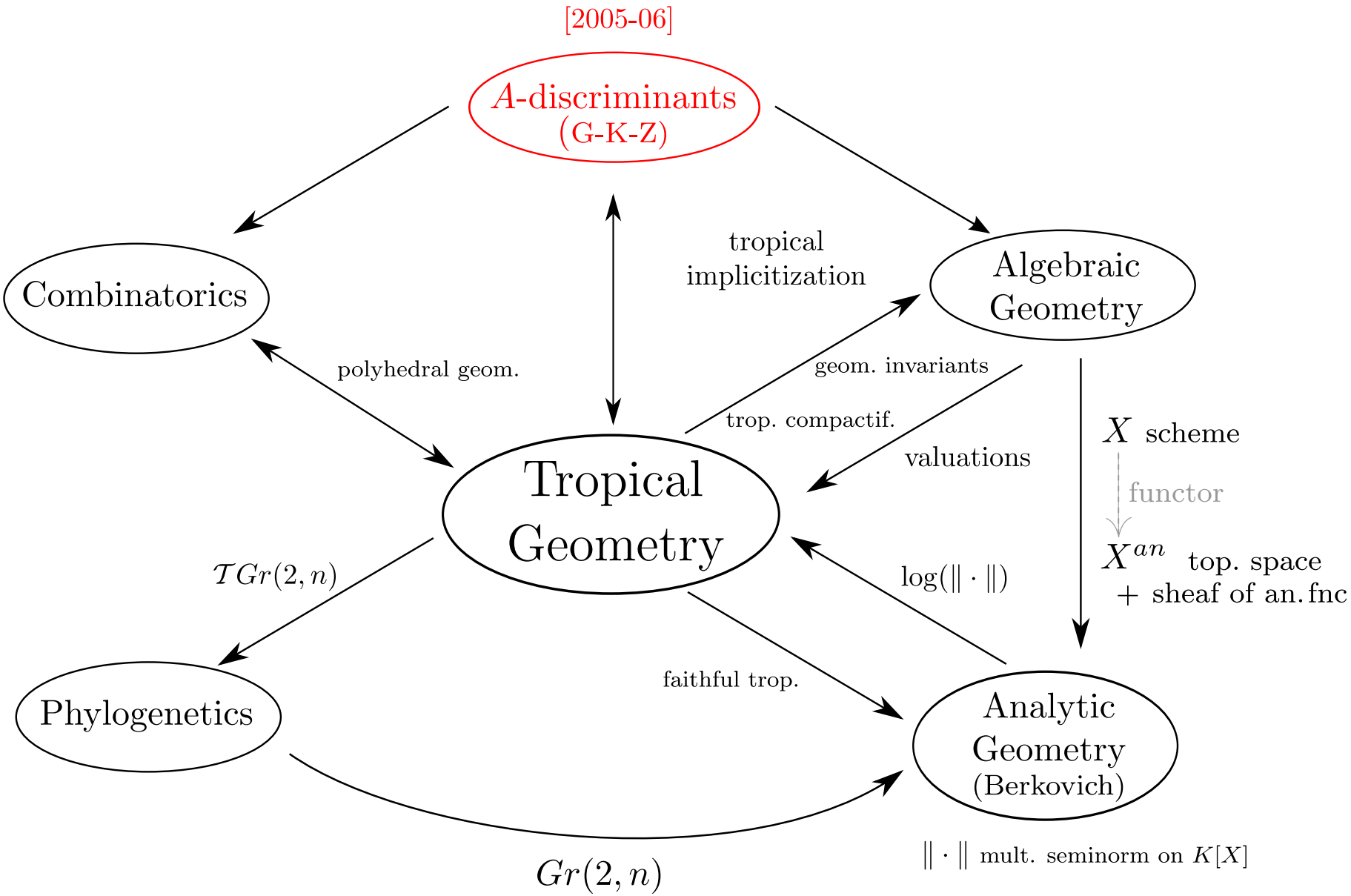
The big Mathematical picture



First stop: Argentina and A-Discriminants



First stop: Argentina and A-Discriminants II

What are A-Discriminants?

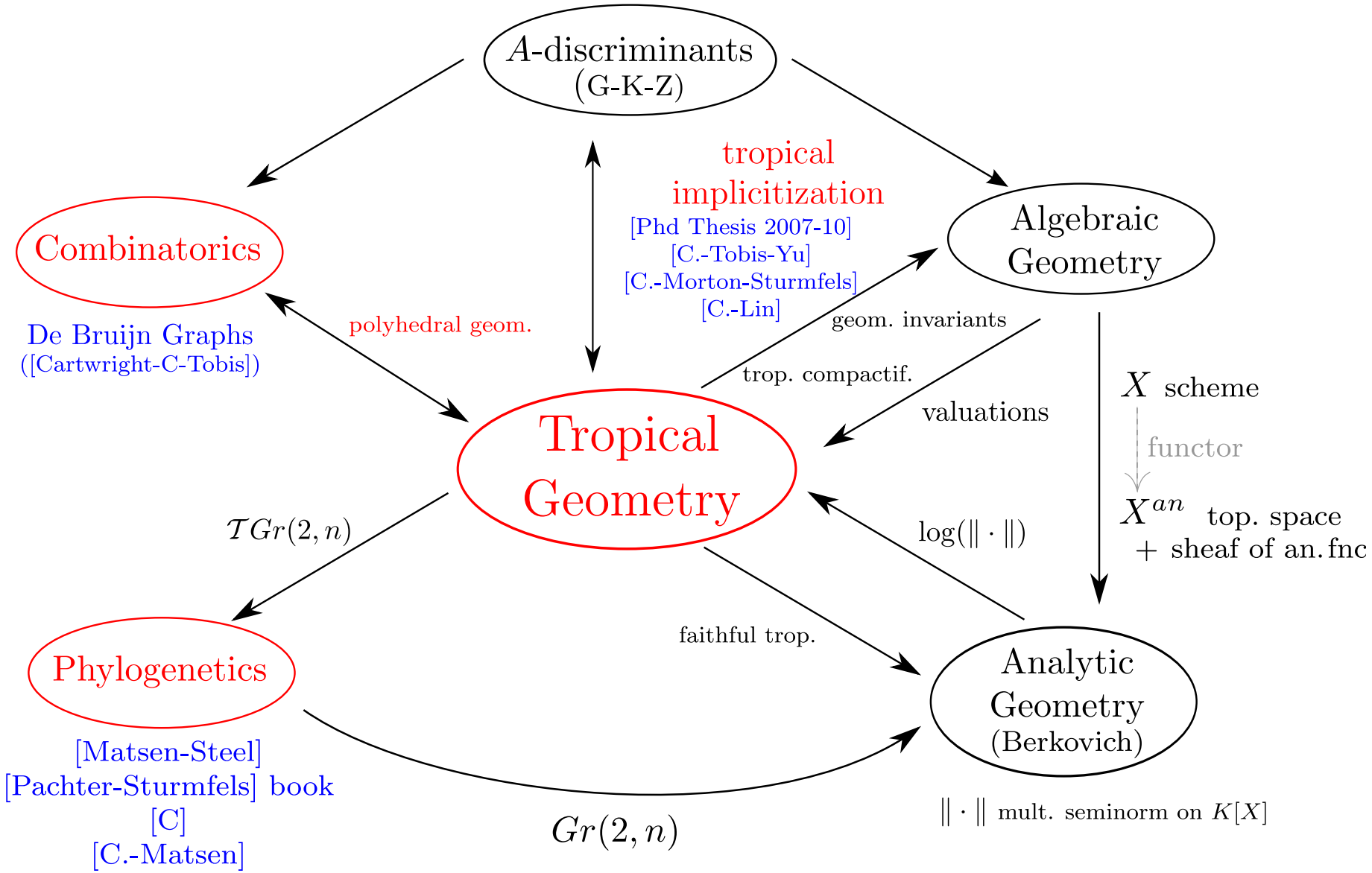
Fix $A \subset \mathbb{Z}^n$ finite $\rightsquigarrow f = \sum_{a \in A} c_a \underline{x}^a \in K[c_a : a \in A][\underline{x}]$.

Want: Polynomial conditions on $(c_a)_{a \in A}$ to decide if f has a singularity in $(K^*)^n$. $\leftrightsquigarrow$ Connections to toric geometry.

Three key events:

- Angra dos Reis conference, March 2005
- Summer School on Res. of Singularities (ICTP, Trieste, Italy), June 2006
- Dickenstein-Feichtner-Sturmfels, *Tropical Discriminants* (2005).

Second stop: UC Berkeley



Second stop I: Phylogenetics

Grad Course on Alg. Statistics and Comp. Biology (Sp. 2008)

•Pachter-Sturmfels, *Algebraic statistics for Computational Biology*, Cambridge U. Press (2005).

• Matsen-Steel, *Phylogenetic mixtures on a single tree can mimic a tree of another topology* (2007).

$$\begin{cases} \text{probability mixtures} = \text{convex combinations (e.g. 2 genes)} \\ \text{tropical analog} = \text{MAX. (take -val)} \end{cases}$$

• C., *Tropical Mixtures of star tree metrics.* (2008).

• C.-Matsen, *Polyhedral Geometry of Phylogenetic Rogue Taxa* (2010) [optimization over the balanced minimum evolution polytope]

Second stop II: Tropical Geometry

- $\overline{\mathbb{R}}_{\text{tr}} := (\mathbb{R} \cup \{-\infty\}, \oplus, \odot)$, $a \oplus b = \max\{a,b\}$, $a \odot b = a+b$.
- Fix $K = \mathbb{C}\{\{t\}\}$ field of Puiseux series, with valuation given by **lowest exp.**, e.g. $\text{val}(t^{-4/3} + 1 + t + \ldots) = -4/3$, $\text{val}(0) = \infty$.

$$\boxed{f(\mathbf{x}) \text{ in } K[x_1^{\pm}, \ldots, x_n^{\pm}] \rightsquigarrow \text{trop}(f)(\boldsymbol{\omega}) \text{ in } \overline{\mathbb{R}}_{\text{tr}}[\omega_1^{\odot\pm}, \ldots, \omega_n^{\odot\pm}]}$$

$$f := \sum_{\alpha} c_\alpha \mathbf{x}^\alpha \mapsto \text{trop}(f)(\boldsymbol{\omega}) := \bigoplus_{\alpha} -\text{val}(c_\alpha) \odot \boldsymbol{\omega}^{\odot\alpha} = \max_{\alpha}\{-\text{val}(c_\alpha) + \langle \alpha, \boldsymbol{\omega} \rangle\}.$$

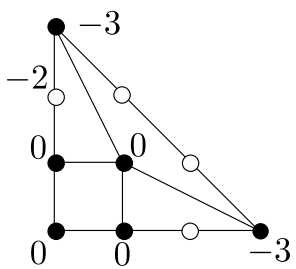
$(f = 0)$ in $(K^*)^n \rightsquigarrow \mathcal{T}f := \{\boldsymbol{\omega} \in \mathbb{R}^n : \text{max in trop}(f)(\boldsymbol{\omega}) \text{is not unique}\}$

$I \subset K[x_1^{\pm}, \ldots, x_n^{\pm}]$ prime dim. $d \rightsquigarrow \mathcal{T}I := \bigcap_{f \in I} \mathcal{T}f \subset \mathbb{R}^n$.

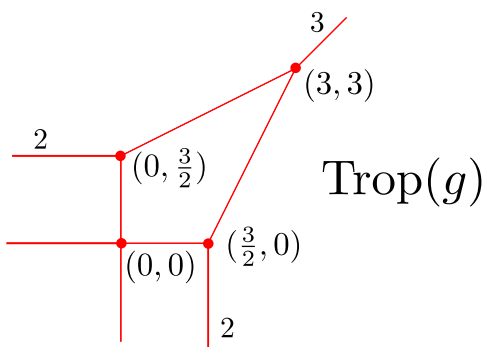
- $\mathcal{T}(I)$ is a pure balance d-dim'l polyhedral complex.

Example: $g = -t^3 x^3 + t^3 y^3 + t^2 y^2 + (4 + t^5)xy + 2x + 7y + (1 + t)$.

Newton subdivision of g



height of $(i,j) = -\text{val}(c_{i,j})$



- tr. mult. $m_{\boldsymbol{\omega}} = \#\{\text{components of in}_{\boldsymbol{\omega}}(I)\}$ (counted with mult.)

Tropical Geometry is a combinatorial shadow of algebraic geometry

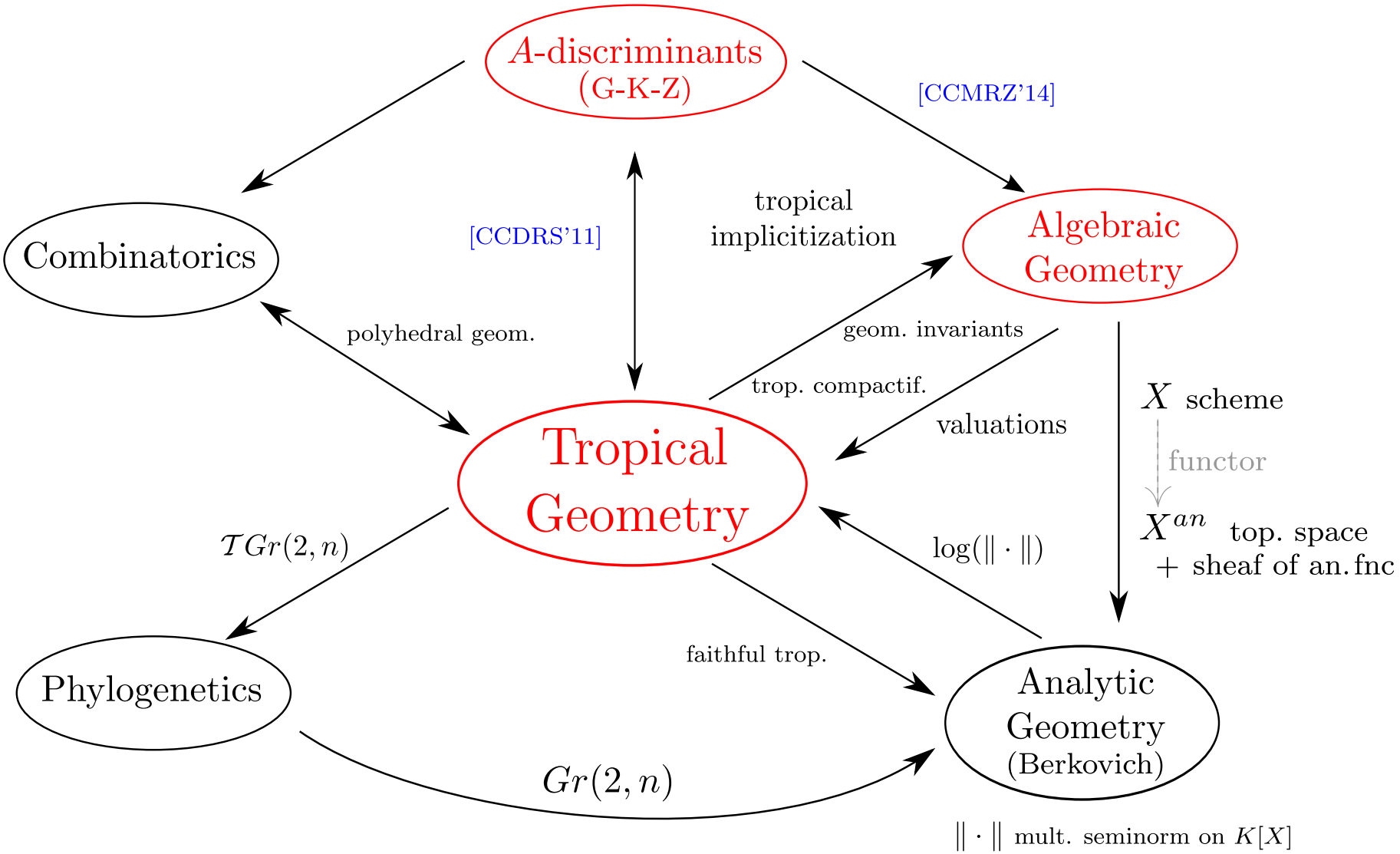
KEY EVENTS:

• Special program on tropical Geometry (MSRI, Berkeley, F. '09)
Encounter the tropical community and all its perspectives.
Met future collaborators/postdoc mentors.

• Tulane Univ. Conference (Nov. 2008).
• MEGA conference, Barcelona U. (June 2009):
C.-Tobis-Yu, *An implicitiz. challenge for binary factor analysis ('09)*
• Sequel: general case with applications to Machine-Learning
C.-Morton-Sturmfels, *Geometry of the restricted Boltzmann machine* (2009) $\longrightarrow$ Contemp. Math. volume.

• Jumbo semester in Algebraic Geometry (MSRI, Berkeley, Sp. '09)
Thesis problem: Geometric tropicalization (Hacking-Keel-Tevelev) and trop. implicitization ["Ask Mr. T. Session."]
Test-case: C.-Lin, *Tropical secant graphs of monomial curves* (2009-2010) $\longrightarrow$ FPSAC Summer 2010.

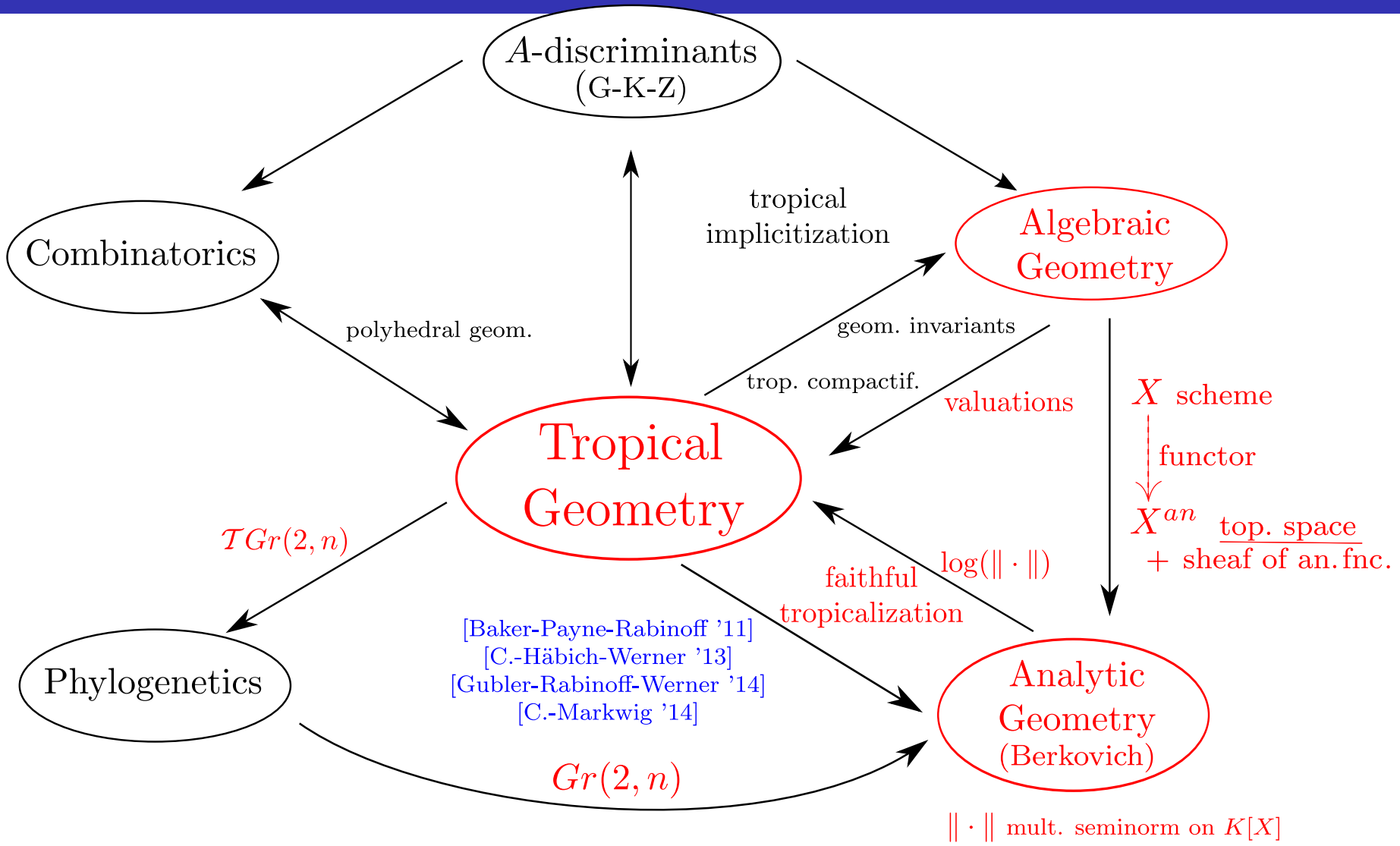
Third stop: Mittag-Leffler Inst. (Special Program Sp. '11)



Cattani-C.-Dickenstein-di Rocco-Sturmfels, *Mixed discriminants* (2011).

Ciliberto-C.-Mella-Ranestad-Zwiernik, *Cremona linearlizations of some classical varieties* (2014).

Fourth stop: NSF and A. v. Humboldt Postdoc Fellowships



C.-Häbich-Werner, *Faithful trop. of the Grassmannian of planes* (2013)

C.-Markwig, *How to repair tropicalizations of plane curves using modifications* (2014).

(1) Your future collaborator might be sitting next to you:

- Attend seminars and Colloquia $\longrightarrow$ R. Vakil 3 messages rule.
- Register for `arXiv` mailing-list: know what's going on!
- Regional meetings - joint seminars with local institutions
- **Special semester programs:** don't miss them!
- **Don't be shy:** let people know you are visiting town...

Talk to people: learn from your peers, colleagues and coparticipants.

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(2) Written communication skills:

- Papers: **short is always better!**. Have an audience in mind
- Aim for pleasant reading, learn from the masters!
- Work on the text and the math *at the same time*.
- Grant proposals / job applications: **start early!** (1-3 months).
- Share a final draft to many people (general math audience).
- `Mathjobs`: apply early! and let people know you applied.
- Ideal letter writers: senior & outside your circle $\longrightarrow$ broad vision.

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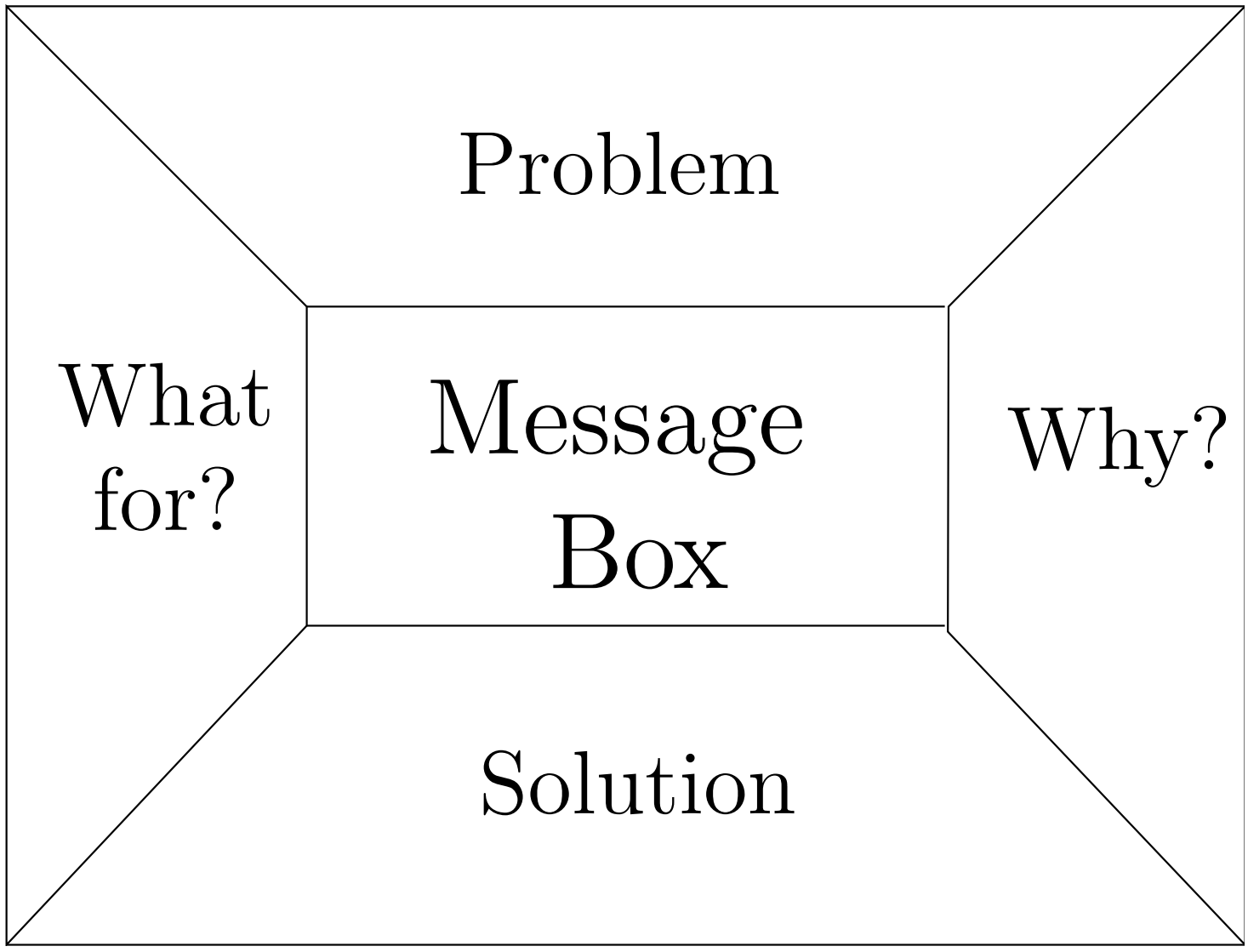
(3) Verbal communication skills:

- practice, practice, practice! $\longrightarrow$ A critical audience is the best!
- 10 minute lecture on your current research: the message box.


Problem
What for?
Message Box
Why?
Solution