6. $\int\left(\sqrt{x^{3}}+\sqrt[3]{x^{2}}\right) d x=\int\left(x^{3 / 2}+x^{2 / 3}\right) d x=\frac{x^{5 / 2}}{5 / 2}+\frac{x^{5 / 3}}{5 / 3}+C=\frac{2}{5} x^{5 / 2}+\frac{3}{5} x^{5 / 3}+C$
7. $\int\left(x^{2}+1+\frac{1}{x^{2}+1}\right) d x=\frac{x^{3}}{3}+x+\tan ^{-1} x+C$
8. $\int\left(1+\tan ^{2} \alpha\right) d \alpha=\int \sec ^{2} \alpha d \alpha=\tan \alpha+C$
9. $\int_{1}^{2} \frac{(x-1)^{3}}{x^{2}} d x=\int_{1}^{2} \frac{x^{3}-3 x^{2}+3 x-1}{x^{2}} d x=\int_{1}^{2}\left(x-3+\frac{3}{x}-\frac{1}{x^{2}}\right) d x=\left[\frac{1}{2} x^{2}-3 x+3 \ln |x|+\frac{1}{x}\right]_{1}^{2}$

$$
=\left(2-6+3 \ln 2+\frac{1}{2}\right)-\left(\frac{1}{2}-3+0+1\right)=3 \ln 2-2
$$

49. $A=\int_{0}^{2}\left(2 y-y^{2}\right) d y=\left[y^{2}-\frac{1}{3} y^{3}\right]_{0}^{2}=\left(4-\frac{8}{3}\right)-0=\frac{4}{3}$
50. (a) Displacement $=\int_{1}^{6}\left(t^{2}-2 t-8\right) d t=\left[\frac{1}{3} t^{3}-t^{2}-8 t\right]_{1}^{6}=(72-36-48)-\left(\frac{1}{3}-1-8\right)=-\frac{10}{3} \mathrm{~m}$
(b) Distance traveled $=\int_{1}^{6}\left|t^{2}-2 t-8\right| d t=\int_{1}^{6}|(t-4)(t+2)| d t$

$$
\begin{aligned}
& =\int_{1}^{4}\left(-t^{2}+2 t+8\right) d t+\int_{4}^{6}\left(t^{2}-2 t-8\right) d t=\left[-\frac{1}{3} t^{3}+t^{2}+8 t\right]_{1}^{4}+\left[\frac{1}{3} t^{3}-t^{2}-8 t\right]_{4}^{6} \\
& =\left(-\frac{64}{3}+16+32\right)-\left(-\frac{1}{3}+1+8\right)+(72-36-48)-\left(\frac{64}{3}-16-32\right)=\frac{98}{3} \mathrm{~m}
\end{aligned}
$$

32. Let $u=\ln x$. Then $d u=(1 / x) d x$, so $\int \frac{\sin (\ln x)}{x} d x=\int \sin u d u=-\cos u+C=-\cos (\ln x)+C$.
33. Let $u=x^{2}$. Then $d u=2 x d x$, so $\int \frac{x}{1+x^{4}} d x=\int \frac{\frac{1}{2} d u}{1+u^{2}}=\frac{1}{2} \tan ^{-1} u+C=\frac{1}{2} \tan ^{-1}\left(x^{2}\right)+C$.
34. Let $u=1+2 x$, so $x=\frac{1}{2}(u-1)$ and $d u=2 d x$. When $x=0, u=1$; when $x=4, u=9$. Thus,

$$
\begin{aligned}
\int_{0}^{4} \frac{x d x}{\sqrt{1+2 x}} & =\int_{1}^{9} \frac{\frac{1}{2}(u-1)}{\sqrt{u}} \frac{d u}{2}=\frac{1}{4} \int_{1}^{9}\left(u^{1 / 2}-u^{-1 / 2}\right) d u=\frac{1}{4}\left[\frac{2}{3} u^{3 / 2}-2 u^{1 / 2}\right]_{1}^{9}=\frac{1}{4} \cdot \frac{2}{3}\left[u^{3 / 2}-3 u^{1 / 2}\right]_{1}^{9} \\
& =\frac{1}{6}[(27-9)-(1-3)]=\frac{20}{6}=\frac{10}{3}
\end{aligned}
$$

86. Let $u=x^{2}$. Then $d u=2 x d x$, so $\int_{0}^{3} x f\left(x^{2}\right) d x=\int_{0}^{9} f(u)\left(\frac{1}{2} d u\right)=\frac{1}{2} \int_{0}^{9} f(u) d u=\frac{1}{2}(4)=2$.
