

7. No, the curve is not the graph of a function because a vertical line intersects the curve more than once. Hence, the curve fails the Vertical Line Test.
8. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-2, 2]$ and the range is $[-1, 2]$.
9. Yes, the curve is the graph of a function because it passes the Vertical Line Test. The domain is $[-3, 2]$ and the range is $[-3, -2) \cup [-1, 3]$.
10. No, the curve is not the graph of a function since for $x = 0, \pm 1,$ and $\pm 2,$ there are infinitely many points on the curve.

25. $f(x) = 3x^2 - x + 2.$

$$f(2) = 3(2)^2 - 2 + 2 = 12 - 2 + 2 = 12.$$

$$f(-2) = 3(-2)^2 - (-2) + 2 = 12 + 2 + 2 = 16.$$

$$f(a) = 3a^2 - a + 2.$$

$$f(-a) = 3(-a)^2 - (-a) + 2 = 3a^2 + a + 2.$$

$$f(a+1) = 3(a+1)^2 - (a+1) + 2 = 3(a^2 + 2a + 1) - a - 1 + 2 = 3a^2 + 6a + 3 - a + 1 = 3a^2 + 5a + 4.$$

$$2f(a) = 2 \cdot f(a) = 2(3a^2 - a + 2) = 6a^2 - 2a + 4.$$

$$f(2a) = 3(2a)^2 - (2a) + 2 = 3(4a^2) - 2a + 2 = 12a^2 - 2a + 2.$$

$$f(a^2) = 3(a^2)^2 - (a^2) + 2 = 3(a^4) - a^2 + 2 = 3a^4 - a^2 + 2.$$

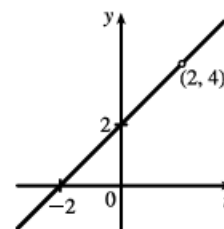
$$\begin{aligned} [f(a)]^2 &= [3a^2 - a + 2]^2 = (3a^2 - a + 2)(3a^2 - a + 2) \\ &= 9a^4 - 3a^3 + 6a^2 - 3a^3 + a^2 - 2a + 6a^2 - 2a + 4 = 9a^4 - 6a^3 + 13a^2 - 4a + 4. \end{aligned}$$

$$f(a+h) = 3(a+h)^2 - (a+h) + 2 = 3(a^2 + 2ah + h^2) - a - h + 2 = 3a^2 + 6ah + 3h^2 - a - h + 2.$$

42. $H(t) = \frac{4-t^2}{2-t} = \frac{(2+t)(2-t)}{2-t},$ so for $t \neq 2, H(t) = 2+t.$ The domain

is $\{t \mid t \neq 2\}.$ So the graph of H is the same as the graph of the function

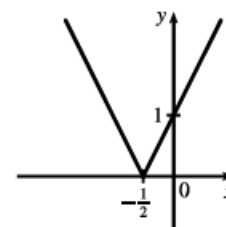
$f(t) = t + 2$ (a line) except for the hole at $(2, 4).$



44.
$$F(x) = |2x + 1| = \begin{cases} 2x + 1 & \text{if } 2x + 1 \geq 0 \\ -(2x + 1) & \text{if } 2x + 1 < 0 \end{cases}$$

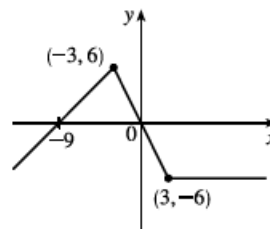
$$= \begin{cases} 2x + 1 & \text{if } x \geq -\frac{1}{2} \\ -2x - 1 & \text{if } x < -\frac{1}{2} \end{cases}$$

The domain is $\mathbb{R},$ or $(-\infty, \infty).$



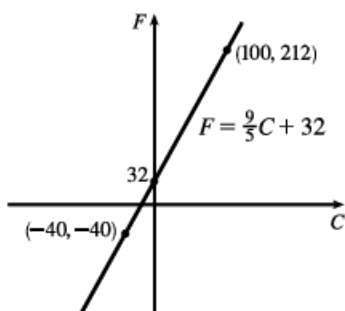
$$50. f(x) = \begin{cases} x + 9 & \text{if } x < -3 \\ -2x & \text{if } |x| \leq 3 \\ -6 & \text{if } x > 3 \end{cases}$$

Note that for $x = -3$, both $x + 9$ and $-2x$ are equal to 6; and for $x = 3$, both $-2x$ and -6 are equal to -6 . The domain is \mathbb{R} .



4. (a) The graph of $y = 3x$ is a line (choice G).
 (b) $y = 3^x$ is an exponential function (choice f).
 (c) $y = x^3$ is an odd polynomial function or power function (choice F).
 (d) $y = \sqrt[3]{x} = x^{1/3}$ is a root function (choice g).

13. (a)



(b) The slope of $\frac{9}{5}$ means that F increases $\frac{9}{5}$ degrees for each increase of 1°C . (Equivalently, F increases by 9 when C increases by 5 and F decreases by 9 when C decreases by 5.) The F -intercept of 32 is the Fahrenheit temperature corresponding to a Celsius temperature of 0.

18. (a) Using d in place of x and C in place of y , we find the slope to be $\frac{C_2 - C_1}{d_2 - d_1} = \frac{460 - 380}{800 - 480} = \frac{80}{320} = \frac{1}{4}$.

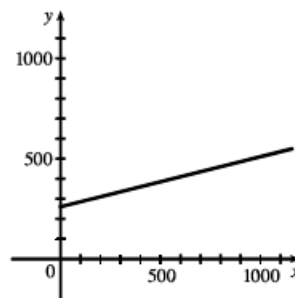
So a linear equation is $C - 460 = \frac{1}{4}(d - 800) \Leftrightarrow C - 460 = \frac{1}{4}d - 200 \Leftrightarrow C = \frac{1}{4}d + 260$.

(b) Letting $d = 1500$ we get $C = \frac{1}{4}(1500) + 260 = 635$.

The cost of driving 1500 miles is \$635.

(d) The y -intercept represents the fixed cost, \$260.

(c)



The slope of the line represents the cost per mile, \$0.25.

(e) A linear function gives a suitable model in this situation because you have fixed monthly costs such as insurance and car payments, as well as costs that increase as you drive, such as gasoline, oil, and tires, and the cost of these for each additional mile driven is a constant.

27. (a) Using a computing device, we obtain a power function $N = cA^b$, where $c \approx 3.1046$ and $b \approx 0.308$.

(b) If $A = 291$, then $N = cA^b \approx 17.8$, so you would expect to find 18 species of reptiles and amphibians on Dominica.

35. $f(x) = x + \frac{1}{x}$, $D = \{x \mid x \neq 0\}$; $g(x) = \frac{x+1}{x+2}$, $D = \{x \mid x \neq -2\}$

$$\begin{aligned} \text{(a)} \quad (f \circ g)(x) &= f(g(x)) = f\left(\frac{x+1}{x+2}\right) = \frac{x+1}{x+2} + \frac{1}{\frac{x+1}{x+2}} = \frac{x+1}{x+2} + \frac{x+2}{x+1} \\ &= \frac{(x+1)(x+1) + (x+2)(x+2)}{(x+2)(x+1)} = \frac{(x^2 + 2x + 1) + (x^2 + 4x + 4)}{(x+2)(x+1)} = \frac{2x^2 + 6x + 5}{(x+2)(x+1)} \end{aligned}$$

Since $g(x)$ is not defined for $x = -2$ and $f(g(x))$ is not defined for $x = -2$ and $x = -1$,

the domain of $(f \circ g)(x)$ is $D = \{x \mid x \neq -2, -1\}$.

$$\text{(b)} \quad (g \circ f)(x) = g(f(x)) = g\left(x + \frac{1}{x}\right) = \frac{\left(x + \frac{1}{x}\right) + 1}{\left(x + \frac{1}{x}\right) + 2} = \frac{\frac{x^2 + 1 + x}{x}}{\frac{x^2 + 1 + 2x}{x}} = \frac{x^2 + x + 1}{x^2 + 2x + 1} = \frac{x^2 + x + 1}{(x+1)^2}$$

Since $f(x)$ is not defined for $x = 0$ and $g(f(x))$ is not defined for $x = -1$,

the domain of $(g \circ f)(x)$ is $D = \{x \mid x \neq -1, 0\}$.

$$\begin{aligned} \text{(c)} \quad (f \circ f)(x) &= f(f(x)) = f\left(x + \frac{1}{x}\right) = \left(x + \frac{1}{x}\right) + \frac{1}{x + \frac{1}{x}} = x + \frac{1}{x} + \frac{1}{\frac{x^2+1}{x}} = x + \frac{1}{x} + \frac{x}{x^2+1} \\ &= \frac{x(x)(x^2+1) + 1(x^2+1) + x(x)}{x(x^2+1)} = \frac{x^4 + x^2 + x^2 + 1 + x^2}{x(x^2+1)} \\ &= \frac{x^4 + 3x^2 + 1}{x(x^2+1)}, \quad D = \{x \mid x \neq 0\} \end{aligned}$$

$$\text{(d)} \quad (g \circ g)(x) = g(g(x)) = g\left(\frac{x+1}{x+2}\right) = \frac{\frac{x+1}{x+2} + 1}{\frac{x+1}{x+2} + 2} = \frac{\frac{x+1+1(x+2)}{x+2}}{\frac{x+1+2(x+2)}{x+2}} = \frac{x+1+x+2}{x+1+2x+4} = \frac{2x+3}{3x+5}$$

Since $g(x)$ is not defined for $x = -2$ and $g(g(x))$ is not defined for $x = -\frac{5}{3}$,

the domain of $(g \circ g)(x)$ is $D = \{x \mid x \neq -2, -\frac{5}{3}\}$.

46. Let $g(t) = \tan t$ and $f(t) = \frac{t}{1+t}$. Then $(f \circ g)(t) = f(g(t)) = f(\tan t) = \frac{\tan t}{1 + \tan t} = u(t)$.

53. (a) Using the relationship $\text{distance} = \text{rate} \cdot \text{time}$ with the radius r as the distance, we have $r(t) = 60t$.

(b) $A = \pi r^2 \Rightarrow (A \circ r)(t) = A(r(t)) = \pi(60t)^2 = 3600\pi t^2$. This formula gives us the extent of the rippled area (in cm^2) at any time t .

56. (a) $d = rt \Rightarrow d(t) = 350t$

(b) There is a Pythagorean relationship involving the legs with lengths d and 1 and the hypotenuse with length s :

$$d^2 + 1^2 = s^2. \text{ Thus, } s(d) = \sqrt{d^2 + 1}.$$

(c) $(s \circ d)(t) = s(d(t)) = s(350t) = \sqrt{(350t)^2 + 1}$

60. If $A(x) = 1.04x$, then

$$(A \circ A)(x) = A(A(x)) = A(1.04x) = 1.04(1.04x) = (1.04)^2x,$$

$$(A \circ A \circ A)(x) = A((A \circ A)(x)) = A((1.04)^2x) = 1.04(1.04)^2x = (1.04)^3x, \text{ and}$$

$$(A \circ A \circ A \circ A)(x) = A((A \circ A \circ A)(x)) = A((1.04)^3x) = 1.04(1.04)^3x = (1.04)^4x.$$

These compositions represent the amount of the investment after 2, 3, and 4 years.

Based on this pattern, when we compose n copies of A , we get the formula $\underbrace{(A \circ A \circ \dots \circ A)}_{n \text{ A's}}(x) = (1.04)^n x$.

17. (a) To find the equation of the graph that results from shifting the graph of $y = e^x$ 2 units downward, we subtract 2 from the original function to get $y = e^x - 2$.

(b) To find the equation of the graph that results from shifting the graph of $y = e^x$ 2 units to the right, we replace x with $x - 2$ in the original function to get $y = e^{(x-2)}$.

(c) To find the equation of the graph that results from reflecting the graph of $y = e^x$ about the x -axis, we multiply the original function by -1 to get $y = -e^x$.

(d) To find the equation of the graph that results from reflecting the graph of $y = e^x$ about the y -axis, we replace x with $-x$ in the original function to get $y = e^{-x}$.

(e) To find the equation of the graph that results from reflecting the graph of $y = e^x$ about the x -axis and then about the y -axis, we first multiply the original function by -1 (to get $y = -e^x$) and then replace x with $-x$ in this equation to get $y = -e^{-x}$.

24. Suppose the month is February. Your payment on the 28th day would be $2^{28-1} = 2^{27} = 134,217,728$ cents, or \$1,342,177.28. Clearly, the second method of payment results in a larger amount for any month.

29. (a) Fifteen hours represents 5 doubling periods (one doubling period is three hours). $100 \cdot 2^5 = 3200$

(b) In t hours, there will be $t/3$ doubling periods. The initial population is 100, so the population y at time t is $y = 100 \cdot 2^{t/3}$.

(c) $t = 20 \Rightarrow y = 100 \cdot 2^{20/3} \approx 10,159$

(d) We graph $y_1 = 100 \cdot 2^{x/3}$ and $y_2 = 50,000$. The two curves intersect at $x \approx 26.9$, so the population reaches 50,000 in about 26.9 hours.

