21. $y = f(x) = 1 + \sqrt{2+3x}\quad (y \geq 1) \quad \Rightarrow \quad y - 1 = \sqrt{2+3x} \quad \Rightarrow \quad (y-1)^2 = 2 + 3x \quad \Rightarrow \quad (y-1)^2 - 2 = 3x \quad \Rightarrow$ $x = \frac{1}{3}(y-1)^2 - \frac{2}{3}$. Interchange x and y: $y = \frac{1}{3}(x-1)^2 - \frac{2}{3}$. So $f^{-1}(x) = \frac{1}{3}(x-1)^2 - \frac{2}{3}$. Note that the domain of f^{-1} is $x \geq 1$.

22. $y = f(x) = \dfrac{4x-1}{2x+3} \quad \Rightarrow \quad y(2x+3) = 4x - 1 \quad \Rightarrow \quad 2xy + 3y = 4x - 1 \quad \Rightarrow \quad 3y + 1 = 4x - 2xy \quad \Rightarrow$ $3y + 1 = (4-2y)x \quad \Rightarrow \quad x = \dfrac{3y+1}{4-2y}$. Interchange x and y: $y = \dfrac{3x+1}{4-2x}$. So $f^{-1}(x) = \dfrac{3x+1}{4-2x}$.

61. (a) $n = f(t) = 100 \cdot 2^{t/3} \quad \Rightarrow \quad \dfrac{n}{100} = 2^{t/3} \quad \Rightarrow \quad \log_2\left(\dfrac{n}{100}\right) = \dfrac{t}{3} \quad \Rightarrow \quad t = 3\log_2\left(\dfrac{n}{100}\right)$. Using formula (10), we can write this as $t = f^{-1}(n) = 3 \cdot \dfrac{\ln(n/100)}{\ln 2}$. This function tells us how long it will take to obtain n bacteria (given the number n).

(b) $n = 50{,}000 \quad \Rightarrow \quad t = f^{-1}(50{,}000) = 3 \cdot \dfrac{\ln\left(\frac{50{,}000}{100}\right)}{\ln 2} = 3\left(\dfrac{\ln 500}{\ln 2}\right) \approx 26.9$ hours

75. $g(x) = \sin^{-1}(3x+1)$.

Domain $(g) = \{x \mid -1 \leq 3x + 1 \leq 1\} = \{x \mid -2 \leq 3x \leq 0\} = \left\{x \mid -\frac{2}{3} \leq x \leq 0\right\} = \left[-\frac{2}{3}, 0\right]$.

Range $(g) = \left\{y \mid -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}\right\} = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

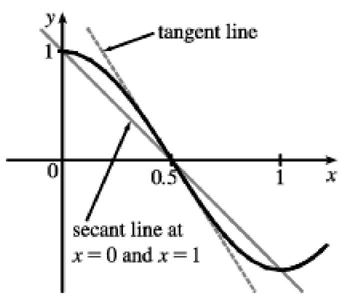
4. (a) $y = \cos \pi x$, $P(0.5, 0)$

	x	Q	m_{PQ}
(i)	0	$(0, 1)$	-2
(ii)	0.4	$(0.4, 0.309017)$	-3.090170
(iii)	0.49	$(0.49, 0.031411)$	-3.141076
(iv)	0.499	$(0.499, 0.003142)$	-3.141587
(v)	1	$(1, -1)$	-2
(vi)	0.6	$(0.6, -0.309017)$	-3.090170
(vii)	0.51	$(0.51, -0.031411)$	-3.141076
(viii)	0.501	$(0.501, -0.003142)$	-3.141587

(b) The slope appears to be $-\pi$.

(c) $y - 0 = -\pi(x - 0.5)$ or $y = -\pi x + \frac{1}{2}\pi$.

(d)



8. (a) (i) $s = s(t) = 2\sin \pi t + 3\cos \pi t$. On the interval $[1, 2]$, $v_{\text{ave}} = \dfrac{s(2) - s(1)}{2 - 1} = \dfrac{3 - (-3)}{1} = 6$ cm/s.

(ii) On the interval $[1, 1.1]$, $v_{\text{ave}} = \dfrac{s(1.1) - s(1)}{1.1 - 1} \approx \dfrac{-3.471 - (-3)}{0.1} = -4.71$ cm/s.

(iii) On the interval $[1, 1.01]$, $v_{\text{ave}} = \dfrac{s(1.01) - s(1)}{1.01 - 1} \approx \dfrac{-3.0613 - (-3)}{0.01} = -6.13$ cm/s.

(iv) On the interval $[1, 1.001]$, $v_{\text{ave}} = \dfrac{s(1.001) - s(1)}{1.001 - 1} \approx \dfrac{-3.00627 - (-3)}{1.001 - 1} = -6.27$ cm/s.

(b) The instantaneous velocity of the particle when $t = 1$ appears to be about -6.3 cm/s.

2. As x approaches 1 from the left, $f(x)$ approaches 3; and as x approaches 1 from the right, $f(x)$ approaches 7. No, the limit does not exist because the left- and right-hand limits are different.

6. (a) $h(x)$ approaches 4 as x approaches -3 from the left, so $\lim\limits_{x\to -3^-} h(x) = 4$.

(b) $h(x)$ approaches 4 as x approaches -3 from the right, so $\lim\limits_{x\to -3^+} h(x) = 4$.

(c) $\lim\limits_{x\to -3} h(x) = 4$ because the limits in part (a) and part (b) are equal.

(d) $h(-3)$ is not defined, so it doesn't exist.

(e) $h(x)$ approaches 1 as x approaches 0 from the left, so $\lim\limits_{x\to 0^-} h(x) = 1$.

(f) $h(x)$ approaches -1 as x approaches 0 from the right, so $\lim\limits_{x\to 0^+} h(x) = -1$.

(g) $\lim\limits_{x\to 0} h(x)$ does not exist because the limits in part (e) and part (f) are not equal.

(h) $h(0) = 1$ since the point $(0, 1)$ is on the graph of h.

(i) Since $\lim\limits_{x\to 2^-} h(x) = 2$ and $\lim\limits_{x\to 2^+} h(x) = 2$, we have $\lim\limits_{x\to 2} h(x) = 2$.

(j) $h(2)$ is not defined, so it doesn't exist.

(k) $h(x)$ approaches 3 as x approaches 5 from the right, so $\lim\limits_{x\to 5^+} h(x) = 3$.

(l) $h(x)$ does not approach any one number as x approaches 5 from the left, so $\lim\limits_{x\to 5^-} h(x)$ does not exist.

7. (a) $\lim\limits_{t\to 0^-} g(t) = -1$

(b) $\lim\limits_{t\to 0^+} g(t) = -2$

(c) $\lim\limits_{t\to 0} g(t)$ does not exist because the limits in part (a) and part (b) are not equal.

(d) $\lim\limits_{t\to 2^-} g(t) = 2$

(e) $\lim\limits_{t\to 2^+} g(t) = 0$

(f) $\lim\limits_{t\to 2} g(t)$ does not exist because the limits in part (d) and part (e) are not equal.

(g) $g(2) = 1$

(h) $\lim\limits_{t\to 4} g(t) = 3$

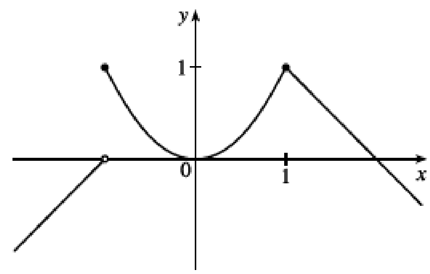
9. (a) $\lim_{x\to -7} f(x) = -\infty$ (b) $\lim_{x\to -3} f(x) = \infty$ (c) $\lim_{x\to 0} f(x) = \infty$

(d) $\lim_{x\to 6^-} f(x) = -\infty$ (e) $\lim_{x\to 6^+} f(x) = \infty$

(f) The equations of the vertical asymptotes are $x = -7$, $x = -3$, $x = 0$, and $x = 6$.

11. From the graph of

$$f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \le x < 1, \\ 2 - x & \text{if } x \ge 1 \end{cases}$$



we see that $\lim_{x\to a} f(x)$ exists for all a except $a = -1$. Notice that the right and left limits are different at $a = -1$.

1. (a) $\lim_{x\to 2} [f(x) + 5g(x)] = \lim_{x\to 2} f(x) + \lim_{x\to 2} [5g(x)]$ [Limit Law 1]

$= \lim_{x\to 2} f(x) + 5 \lim_{x\to 2} g(x)$ [Limit Law 3]

$= 4 + 5(-2) = -6$

(b) $\lim_{x\to 2} [g(x)]^3 = \left[\lim_{x\to 2} g(x)\right]^3$ [Limit Law 6]

$= (-2)^3 = -8$

(c) $\lim_{x\to 2} \sqrt{f(x)} = \sqrt{\lim_{x\to 2} f(x)}$ [Limit Law 11]

$= \sqrt{4} = 2$

(d) $\lim_{x\to 2} \dfrac{3f(x)}{g(x)} = \dfrac{\lim_{x\to 2} [3f(x)]}{\lim_{x\to 2} g(x)}$ [Limit Law 5]

$= \dfrac{3 \lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)}$ [Limit Law 3]

$= \dfrac{3(4)}{-2} = -6$

(e) Because the limit of the denominator is 0, we can't use Limit Law 5. The given limit, $\lim_{x\to 2} \dfrac{g(x)}{h(x)}$, does not exist because the denominator approaches 0 while the numerator approaches a nonzero number.

(f) $\lim_{x\to 2} \dfrac{g(x)\,h(x)}{f(x)} = \dfrac{\lim_{x\to 2} [g(x)\,h(x)]}{\lim_{x\to 2} f(x)}$ [Limit Law 5]

$= \dfrac{\lim_{x\to 2} g(x) \cdot \lim_{x\to 2} h(x)}{\lim_{x\to 2} f(x)}$ [Limit Law 4]

$= \dfrac{-2 \cdot 0}{4} = 0$

10. (a) The left-hand side of the equation is not defined for $x = 2$, but the right-hand side is.

(b) Since the equation holds for all $x \neq 2$, it follows that both sides of the equation approach the same limit as $x \to 2$, just as in Example 3. Remember that in finding $\lim_{x\to a} f(x)$, we never consider $x = a$.

26. $\lim_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2 + t}\right) = \lim_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t(t+1)}\right) = \lim_{t\to 0} \dfrac{t + 1 - 1}{t(t+1)} = \lim_{t\to 0} \dfrac{1}{t+1} = \dfrac{1}{0+1} = 1$

38. We have $\lim_{x\to 1}(2x) = 2(1) = 2$ and $\lim_{x\to 1}(x^4 - x^2 + 2) = 1^4 - 1^2 + 2 = 2$. Since $2x \le g(x) \le x^4 - x^2 + 2$ for all x, $\lim_{x\to 1} g(x) = 2$ by the Squeeze Theorem.

41. $|x-3| = \begin{cases} x-3 & \text{if } x-3 \ge 0 \\ -(x-3) & \text{if } x-3 < 0 \end{cases} = \begin{cases} x-3 & \text{if } x \ge 3 \\ 3-x & \text{if } x < 3 \end{cases}$

Thus, $\lim_{x\to 3^+}(2x + |x-3|) = \lim_{x\to 3^+}(2x + x - 3) = \lim_{x\to 3^+}(3x-3) = 3(3) - 3 = 6$ and $\lim_{x\to 3^-}(2x + |x-3|) = \lim_{x\to 3^-}(2x + 3 - x) = \lim_{x\to 3^-}(x+3) = 3 + 3 = 6$. Since the left and right limits are equal, $\lim_{x\to 3}(2x + |x-3|) = 6$.

44. Since $|x| = -x$ for $x < 0$, we have $\lim_{x\to -2} \dfrac{2-|x|}{2+x} = \lim_{x\to -2} \dfrac{2-(-x)}{2+x} = \lim_{x\to -2} \dfrac{2+x}{2+x} = \lim_{x\to -2} 1 = 1$.

59. Observe that $0 \le f(x) \le x^2$ for all x, and $\lim_{x\to 0} 0 = 0 = \lim_{x\to 0} x^2$. So, by the Squeeze Theorem, $\lim_{x\to 0} f(x) = 0$.