11. If f and g are continuous and $g(2) = 6$, then $\lim_{x\to 2}[3f(x) + f(x)\,g(x)] = 36 \quad\Rightarrow$

$3\lim_{x\to 2} f(x) + \lim_{x\to 2} f(x) \cdot \lim_{x\to 2} g(x) = 36 \quad\Rightarrow\quad 3f(2) + f(2)\cdot 6 = 36 \quad\Rightarrow\quad 9f(2) = 36 \quad\Rightarrow\quad f(2) = 4.$

14. $\lim_{t\to 1} h(t) = \lim_{t\to 1} \dfrac{2t - 3t^2}{1 + t^3} = \dfrac{\lim_{t\to 1}(2t - 3t^2)}{\lim_{t\to 1}(1 + t^3)} = \dfrac{2\lim_{t\to 1} t - 3\lim_{t\to 1} t^2}{\lim_{t\to 1} 1 + \lim_{t\to 1} t^3} = \dfrac{2(1) - 3(1)^2}{1 + (1)^3} = \dfrac{-1}{2} = h(1).$

By the definition of continuity, h is continuous at $a = 1$.

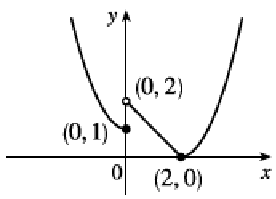
16. For $a < 3$, we have

$$\begin{aligned}
\lim_{x\to a} g(x) &= \lim_{x\to a} 2\sqrt{3 - x} \\
&= 2\lim_{x\to a}\sqrt{3 - x} && \text{[Limit Law 3]} \\
&= 2\sqrt{\lim_{x\to a}(3 - x)} && [11] \\
&= 2\sqrt{\lim_{x\to a} 3 - \lim_{x\to a} x} && [2] \\
&= 2\sqrt{3 - a} && \text{[7 and 8]} \\
&= g(a)
\end{aligned}$$

So g is continuous at $x = a$ for every a in $(-\infty, 3)$. Also, $\lim_{x\to 3^-} g(x) = 0 = g(3)$, so g is continuous from the left at 3.

Thus, g is continuous on $(-\infty, 3]$.

41. $f(x) = \begin{cases} 1 + x^2 & \text{if } x \le 0 \\ 2 - x & \text{if } 0 < x \le 2 \\ (x - 2)^2 & \text{if } x > 2 \end{cases}$



f is continuous on $(-\infty, 0)$, $(0, 2)$, and $(2, \infty)$ since it is a polynomial on each of these intervals. Now $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}(1 + x^2) = 1$ and $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}(2 - x) = 2$, so f is discontinuous at 0. Since $f(0) = 1$, f is continuous from the left at 0. Also, $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-}(2 - x) = 0$, $\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+}(x - 2)^2 = 0$, and $f(2) = 0$, so f is continuous at 2. The only number at which f is discontinuous is 0.

46. $f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x < 2 \\ ax^2 - bx + 3 & \text{if } 2 \le x < 3 \\ 2x - a + b & \text{if } x \ge 3 \end{cases}$

At $x = 2$: $\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} \dfrac{x^2-4}{x-2} = \lim\limits_{x\to 2^-} \dfrac{(x+2)(x-2)}{x-2} = \lim\limits_{x\to 2^-} (x+2) = 2+2 = 4$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} (ax^2 - bx + 3) = 4a - 2b + 3$

We must have $4a - 2b + 3 = 4$, or $4a - 2b = 1$ **(1)**.

At $x = 3$: $\lim\limits_{x\to 3^-} f(x) = \lim\limits_{x\to 3^-} (ax^2 - bx + 3) = 9a - 3b + 3$

$\lim\limits_{x\to 3^+} f(x) = \lim\limits_{x\to 3^+} (2x - a + b) = 6 - a + b$

We must have $9a - 3b + 3 = 6 - a + b$, or $10a - 4b = 3$ **(2)**.

Now solve the system of equations by adding -2 times equation **(1)** to equation **(2)**.

$$\begin{aligned} -8a + 4b &= -2 \\ 10a - 4b &= 3 \\ \hline 2a \phantom{{}-4b} &= 1 \end{aligned}$$

So $a = \frac{1}{2}$. Substituting $\frac{1}{2}$ for a in **(1)** gives us $-2b = -1$, so $b = \frac{1}{2}$ as well. Thus, for f to be continuous on $(-\infty, \infty)$, $a = b = \frac{1}{2}$.

52. $f(x) = \sqrt[3]{x} + x - 1$ is continuous on the interval $[0, 1]$, $f(0) = -1$, and $f(1) = 1$. Since $-1 < 0 < 1$, there is a number c in $(0, 1)$ such that $f(c) = 0$ by the Intermediate Value Theorem. Thus, there is a root of the equation $\sqrt[3]{x} + x - 1 = 0$, or $\sqrt[3]{x} = 1 - x$, in the interval $(0, 1)$.

64. $g(x) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ x & \text{if } x \text{ is irrational} \end{cases}$ is continuous at 0. To see why, note that $-|x| \le g(x) \le |x|$, so by the Squeeze Theorem $\lim\limits_{x\to 0} g(x) = 0 = g(0)$. But g is continuous nowhere else. For if $a \ne 0$ and $\delta > 0$, the interval $(a - \delta, a + \delta)$ contains both infinitely many rational and infinitely many irrational numbers. Since $g(a) = 0$ or a, there are infinitely many numbers x with $0 < |x - a| < \delta$ and $|g(x) - g(a)| > |a|/2$. Thus, $\lim\limits_{x\to a} g(x) \ne g(a)$.

3. (a) $\lim\limits_{x\to\infty} f(x) = -2$ (b) $\lim\limits_{x\to-\infty} f(x) = 2$ (c) $\lim\limits_{x\to 1} f(x) = \infty$

(d) $\lim\limits_{x\to 3} f(x) = -\infty$ (e) Vertical: $x = 1$, $x = 3$; horizontal: $y = -2$, $y = 2$

14. $$\lim_{x\to\infty}\sqrt{\frac{12x^3-5x+2}{1+4x^2+3x^3}}=\sqrt{\lim_{x\to\infty}\frac{12x^3-5x+2}{1+4x^2+3x^3}}$$ [Limit Law 11]

$$=\sqrt{\lim_{x\to\infty}\frac{12-5/x^2+2/x^3}{1/x^3+4/x+3}}$$ [divide by x^3]

$$=\sqrt{\frac{\lim_{x\to\infty}(12-5/x^2+2/x^3)}{\lim_{x\to\infty}(1/x^3+4/x+3)}}$$ [Limit Law 5]

$$=\sqrt{\frac{\lim_{x\to\infty}12-\lim_{x\to\infty}(5/x^2)+\lim_{x\to\infty}(2/x^3)}{\lim_{x\to\infty}(1/x^3)+\lim_{x\to\infty}(4/x)+\lim_{x\to\infty}3}}$$ [Limit Laws 1 and 2]

$$=\sqrt{\frac{12-5\lim_{x\to\infty}(1/x^2)+2\lim_{x\to\infty}(1/x^3)}{\lim_{x\to\infty}(1/x^3)+4\lim_{x\to\infty}(1/x)+3}}$$ [Limit Laws 7 and 3]

$$=\sqrt{\frac{12-5(0)+2(0)}{0+4(0)+3}}$$ [Theorem 5 of Section 2.5]

$$=\sqrt{\frac{12}{3}}=\sqrt{4}=2$$

27. $$\lim_{x\to\infty}\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)=\lim_{x\to\infty}\frac{\left(\sqrt{x^2+ax}-\sqrt{x^2+bx}\right)\left(\sqrt{x^2+ax}+\sqrt{x^2+bx}\right)}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}$$

$$=\lim_{x\to\infty}\frac{(x^2+ax)-(x^2+bx)}{\sqrt{x^2+ax}+\sqrt{x^2+bx}}=\lim_{x\to\infty}\frac{[(a-b)x]/x}{\left(\sqrt{x^2+ax}+\sqrt{x^2+bx}\right)/\sqrt{x^2}}$$

$$=\lim_{x\to\infty}\frac{a-b}{\sqrt{1+a/x}+\sqrt{1+b/x}}=\frac{a-b}{\sqrt{1+0}+\sqrt{1+0}}=\frac{a-b}{2}$$

37. Since $-1\le\cos x\le 1$ and $e^{-2x}>0$, we have $-e^{-2x}\le e^{-2x}\cos x\le e^{-2x}$. We know that $\lim_{x\to\infty}(-e^{-2x})=0$ and $\lim_{x\to\infty}(e^{-2x})=0$, so by the Squeeze Theorem, $\lim_{x\to\infty}(e^{-2x}\cos x)=0$.

3. (a) (i) Using Definition 1 with $f(x) = 4x - x^2$ and $P(1,3)$,

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 1} \frac{(4x - x^2) - 3}{x - 1} = \lim_{x \to 1} \frac{-(x^2 - 4x + 3)}{x - 1} = \lim_{x \to 1} \frac{-(x-1)(x-3)}{x - 1}$$

$$= \lim_{x \to 1} (3 - x) = 3 - 1 = 2$$

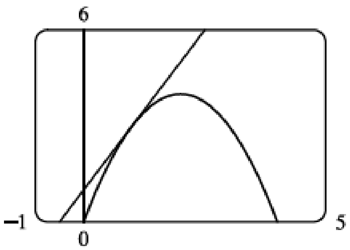
(ii) Using Equation 2 with $f(x) = 4x - x^2$ and $P(1,3)$,

$$m = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h} = \lim_{h \to 0} \frac{\left[4(1+h) - (1+h)^2\right] - 3}{h}$$

$$= \lim_{h \to 0} \frac{4 + 4h - 1 - 2h - h^2 - 3}{h} = \lim_{h \to 0} \frac{-h^2 + 2h}{h} = \lim_{h \to 0} \frac{h(-h+2)}{h} = \lim_{h \to 0} (-h + 2) = 2$$

(b) An equation of the tangent line is $y - f(a) = f'(a)(x - a) \Rightarrow y - f(1) = f'(1)(x - 1) \Rightarrow y - 3 = 2(x - 1)$, or $y = 2x + 1$.

(c)



The graph of $y = 2x + 1$ is tangent to the graph of $y = 4x - x^2$ at the point $(1,3)$. Now zoom in toward the point $(1,3)$ until the parabola and the tangent line are indistiguishable.

8. Using (1) with $f(x) = \dfrac{2x+1}{x+2}$ and $P(1,1)$,

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to 1} \frac{\dfrac{2x+1}{x+2} - 1}{x - 1} = \lim_{x \to 1} \frac{\dfrac{2x + 1 - (x+2)}{x+2}}{x - 1} = \lim_{x \to 1} \frac{x - 1}{(x-1)(x+2)}$$

$$= \lim_{x \to 1} \frac{1}{x+2} = \frac{1}{1+2} = \frac{1}{3}$$

Tangent line: $y - 1 = \frac{1}{3}(x - 1) \quad \Leftrightarrow \quad y - 1 = \frac{1}{3}x - \frac{1}{3} \quad \Leftrightarrow \quad y = \frac{1}{3}x + \frac{2}{3}$

19. For the tangent line $y = 4x - 5$: when $x = 2$, $y = 4(2) - 5 = 3$ and its slope is 4 (the coefficient of x). At the point of tangency, these values are shared with the curve $y = f(x)$; that is, $f(2) = 3$ and $f'(2) = 4$.

39. $v(5) = f'(5) = \displaystyle\lim_{h \to 0} \frac{f(5+h) - f(5)}{h} = \lim_{h \to 0} \frac{[100 + 50(5+h) - 4.9(5+h)^2] - [100 + 50(5) - 4.9(5)^2]}{h}$

$$= \lim_{h \to 0} \frac{(100 + 250 + 50h - 4.9h^2 - 49h - 122.5) - (100 + 250 - 122.5)}{h} = \lim_{h \to 0} \frac{-4.9h^2 + h}{h}$$

$$= \lim_{h \to 0} \frac{h(-4.9h + 1)}{h} = \lim_{h \to 0} (-4.9h + 1) = 1 \text{ m/s}$$

The speed when $t = 5$ is $|1| = 1$ m/s.

50. (a) $f'(8)$ is the rate of change of the quantity of coffee sold with respect to the price per pound when the price is \$8 per pound. The units for $f'(8)$ are pounds/(dollars/pound).

(b) $f'(8)$ is negative since the quantity of coffee sold will decrease as the price charged for it increases. People are generally less willing to buy a product when its price increases.

51. (a) $S'(T)$ is the rate at which the oxygen solubility changes with respect to the water temperature. Its units are $(\text{mg/L})/^\circ\text{C}$.

(b) For $T = 16^\circ\text{C}$, it appears that the tangent line to the curve goes through the points $(0, 14)$ and $(32, 6)$. So $S'(16) \approx \frac{6-14}{32-0} = -\frac{8}{32} = -0.25\ (\text{mg/L})/^\circ\text{C}$. This means that as the temperature increases past 16°C, the oxygen solubility is decreasing at a rate of $0.25\ (\text{mg/L})/^\circ\text{C}$.

3. (a)$'$ = II, since from left to right, the slopes of the tangents to graph (a) start out negative, become 0, then positive, then 0, then negative again. The actual function values in graph II follow the same pattern.

(b)$'$ = IV, since from left to right, the slopes of the tangents to graph (b) start out at a fixed positive quantity, then suddenly become negative, then positive again. The discontinuities in graph IV indicate sudden changes in the slopes of the tangents.

(c)$'$ = I, since the slopes of the tangents to graph (c) are negative for $x < 0$ and positive for $x > 0$, as are the function values of graph I.

(d)$'$ = III, since from left to right, the slopes of the tangents to graph (d) are positive, then 0, then negative, then 0, then positive, then 0, then negative again, and the function values in graph III follow the same pattern.

22. $$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{[m(x+h)+b]-(mx+b)}{h} = \lim_{h\to 0} \frac{mx+mh+b-mx-b}{h}$$
$$= \lim_{h\to 0} \frac{mh}{h} = \lim_{h\to 0} m = m$$

Domain of f = domain of $f' = \mathbb{R}$.

23. $$f'(t) = \lim_{h\to 0} \frac{f(t+h)-f(t)}{h} = \lim_{h\to 0} \frac{\left[5(t+h)-9(t+h)^2\right]-(5t-9t^2)}{h}$$
$$= \lim_{h\to 0} \frac{5t+5h-9(t^2+2th+h^2)-5t+9t^2}{h} = \lim_{h\to 0} \frac{5t+5h-9t^2-18th-9h^2-5t+9t^2}{h}$$
$$= \lim_{h\to 0} \frac{5h-18th-9h^2}{h} = \lim_{h\to 0} \frac{h(5-18t-9h)}{h} = \lim_{h\to 0} (5-18t-9h) = 5-18t$$

Domain of f = domain of $f' = \mathbb{R}$.

26. $g'(t) = \lim\limits_{h \to 0} \dfrac{g(t+h) - g(t)}{h} = \lim\limits_{h \to 0} \dfrac{\dfrac{1}{\sqrt{t+h}} - \dfrac{1}{\sqrt{t}}}{h} = \lim\limits_{h \to 0} \dfrac{\dfrac{\sqrt{t} - \sqrt{t+h}}{\sqrt{t+h}\,\sqrt{t}}}{h} = \lim\limits_{h \to 0} \left(\dfrac{\sqrt{t} - \sqrt{t+h}}{h\,\sqrt{t+h}\,\sqrt{t}} \cdot \dfrac{\sqrt{t} + \sqrt{t+h}}{\sqrt{t} + \sqrt{t+h}} \right)$

$$= \lim_{h \to 0} \frac{t - (t+h)}{h\,\sqrt{t+h}\,\sqrt{t}\,\left(\sqrt{t} + \sqrt{t+h}\right)} = \lim_{h \to 0} \frac{-h}{h\,\sqrt{t+h}\,\sqrt{t}\,\left(\sqrt{t} + \sqrt{t+h}\right)} = \lim_{h \to 0} \frac{-1}{\sqrt{t+h}\,\sqrt{t}\,\left(\sqrt{t} + \sqrt{t+h}\right)}$$

$$= \frac{-1}{\sqrt{t}\,\sqrt{t}\,\left(\sqrt{t} + \sqrt{t}\right)} = \frac{-1}{t\,\left(2\,\sqrt{t}\right)} = -\frac{1}{2t^{3/2}}$$

Domain of g = domain of $g' = (0, \infty)$.

40. f is not differentiable at $x = -1$, because there is a discontinuity there, and at $x = 2$, because the graph has a corner there.