

$$34. y = x^2 e^{-1/x} \Rightarrow y' = x^2 e^{-1/x} \left(\frac{1}{x^2} \right) + e^{-1/x} (2x) = e^{-1/x} + 2x e^{-1/x} = e^{-1/x} (1 + 2x)$$

$$47. y = \cos(x^2) \Rightarrow y' = -\sin(x^2) \cdot 2x = -2x \sin(x^2) \Rightarrow$$

$$y'' = -2x \cos(x^2) \cdot 2x + \sin(x^2) \cdot (-2) = -4x^2 \cos(x^2) - 2 \sin(x^2)$$

$$62. h(x) = \sqrt{4 + 3f(x)} \Rightarrow h'(x) = \frac{1}{2}(4 + 3f(x))^{-1/2} \cdot 3f'(x), \text{ so}$$

$$h'(1) = \frac{1}{2}(4 + 3f(1))^{-1/2} \cdot 3f'(1) = \frac{1}{2}(4 + 3 \cdot 7)^{-1/2} \cdot 3 \cdot 4 = \frac{6}{\sqrt{25}} = \frac{6}{5}$$

65. (a) $u(x) = f(g(x)) \Rightarrow u'(x) = f'(g(x))g'(x)$. So $u'(1) = f'(g(1))g'(1) = f'(3)g'(1)$. To find $f'(3)$, note that f is linear from $(2, 4)$ to $(6, 3)$, so its slope is $\frac{3-4}{6-2} = -\frac{1}{4}$. To find $g'(1)$, note that g is linear from $(0, 6)$ to $(2, 0)$, so its slope is $\frac{0-6}{2-0} = -3$. Thus, $f'(3)g'(1) = (-\frac{1}{4})(-3) = \frac{3}{4}$.

(b) $v(x) = g(f(x)) \Rightarrow v'(x) = g'(f(x))f'(x)$. So $v'(1) = g'(f(1))f'(1) = g'(2)f'(1)$, which does not exist since $g'(2)$ does not exist.

(c) $w(x) = g(g(x)) \Rightarrow w'(x) = g'(g(x))g'(x)$. So $w'(1) = g'(g(1))g'(1) = g'(3)g'(1)$. To find $g'(3)$, note that g is linear from $(2, 0)$ to $(5, 2)$, so its slope is $\frac{2-0}{5-2} = \frac{2}{3}$. Thus, $g'(3)g'(1) = (\frac{2}{3})(-3) = -2$.

1. (a) $\frac{d}{dx}(9x^2 - y^2) = \frac{d}{dx}(1) \Rightarrow 18x - 2y y' = 0 \Rightarrow 2y y' = 18x \Rightarrow y' = \frac{9x}{y}$

(b) $9x^2 - y^2 = 1 \Rightarrow y^2 = 9x^2 - 1 \Rightarrow y = \pm\sqrt{9x^2 - 1}$, so $y' = \pm\frac{1}{2}(9x^2 - 1)^{-1/2}(18x) = \pm\frac{9x}{\sqrt{9x^2 - 1}}$.

(c) From part (a), $y' = \frac{9x}{y} = \frac{9x}{\pm\sqrt{9x^2 - 1}}$, which agrees with part (b).

25. $y \sin 2x = x \cos 2y \Rightarrow y \cdot \cos 2x \cdot 2 + \sin 2x \cdot y' = x(-\sin 2y \cdot 2y') + \cos(2y) \cdot 1 \Rightarrow$

$$\sin 2x \cdot y' + 2x \sin 2y \cdot y' = -2y \cos 2x + \cos 2y \Rightarrow$$

$$y'(\sin 2x + 2x \sin 2y) = -2y \cos 2x + \cos 2y \Rightarrow y' = \frac{-2y \cos 2x + \cos 2y}{\sin 2x + 2x \sin 2y}$$

When $x = \frac{\pi}{2}$ and $y = \frac{\pi}{4}$, we have

$$y' = \frac{(-\pi/2)(-1) + 0}{0 + \pi \cdot 1} = \frac{\pi/2}{\pi} = \frac{1}{2}, \text{ so an equation of the tangent line is } y - \frac{\pi}{4} = \frac{1}{2}(x - \frac{\pi}{2}), \text{ or } y = \frac{1}{2}x.$$

28. $x^2 + 2xy - y^2 + x = 2 \Rightarrow 2x + 2(x y' + y \cdot 1) - 2y y' + 1 = 0 \Rightarrow 2x y' - 2y y' = -2x - 2y - 1 \Rightarrow$

$$y'(2x - 2y) = -2x - 2y - 1 \Rightarrow y' = \frac{-2x - 2y - 1}{2x - 2y}$$

When $x = 1$ and $y = 2$, we have

$$y' = \frac{-2 - 4 - 1}{2 - 4} = \frac{-7}{-2} = \frac{7}{2}, \text{ so an equation of the tangent line is } y - 2 = \frac{7}{2}(x - 1) \text{ or } y = \frac{7}{2}x - \frac{3}{2}.$$

$$32. y^2(y^2 - 4) = x^2(x^2 - 5) \Rightarrow y^4 - 4y^2 = x^4 - 5x^2 \Rightarrow 4y^3 y' - 8y y' = 4x^3 - 10x.$$

When $x = 0$ and $y = -2$, we have $-32y' + 16y' = 0 \Rightarrow -16y' = 0 \Rightarrow y' = 0$, so an equation of the tangent line is $y + 2 = 0(x - 0)$ or $y = -2$.

$$45. \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \Rightarrow \frac{2x}{a^2} - \frac{2yy'}{b^2} = 0 \Rightarrow y' = \frac{b^2 x}{a^2 y} \Rightarrow \text{an equation of the tangent line at } (x_0, y_0) \text{ is}$$

$y - y_0 = \frac{b^2 x_0}{a^2 y_0} (x - x_0)$. Multiplying both sides by $\frac{y_0}{b^2}$ gives $\frac{y_0 y}{b^2} - \frac{y_0^2}{b^2} = \frac{x_0 x}{a^2} - \frac{x_0^2}{a^2}$. Since (x_0, y_0) lies on the hyperbola,

$$\text{we have } \frac{x_0 x}{a^2} - \frac{y_0 y}{b^2} = \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1.$$

$$75. x^2 y^2 + xy = 2 \Rightarrow x^2 \cdot 2yy' + y^2 \cdot 2x + x \cdot y' + y \cdot 1 = 0 \Leftrightarrow y'(2x^2 y + x) = -2xy^2 - y \Leftrightarrow$$

$$y' = -\frac{2xy^2 + y}{2x^2 y + x}. \text{ So } -\frac{2xy^2 + y}{2x^2 y + x} = -1 \Leftrightarrow 2xy^2 + y = 2x^2 y + x \Leftrightarrow y(2xy + 1) = x(2xy + 1) \Leftrightarrow$$

$$y(2xy + 1) - x(2xy + 1) = 0 \Leftrightarrow (2xy + 1)(y - x) = 0 \Leftrightarrow xy = -\frac{1}{2} \text{ or } y = x. \text{ But } xy = -\frac{1}{2} \Rightarrow$$

$$x^2 y^2 + xy = \frac{1}{4} - \frac{1}{2} \neq 2, \text{ so we must have } x = y. \text{ Then } x^2 y^2 + xy = 2 \Rightarrow x^4 + x^2 = 2 \Leftrightarrow x^4 + x^2 - 2 = 0 \Leftrightarrow$$

$$(x^2 + 2)(x^2 - 1) = 0. \text{ So } x^2 = -2, \text{ which is impossible, or } x^2 = 1 \Leftrightarrow x = \pm 1. \text{ Since } x = y, \text{ the points on the curve}$$

where the tangent line has a slope of -1 are $(-1, -1)$ and $(1, 1)$.

$$4. f(x) = \ln(\sin^2 x) = \ln(\sin x)^2 = 2 \ln |\sin x| \Rightarrow f'(x) = 2 \cdot \frac{1}{\sin x} \cdot \cos x = 2 \cot x$$

$$15. F(s) = \ln \ln s \Rightarrow F'(s) = \frac{1}{\ln s} \frac{d}{ds} \ln s = \frac{1}{\ln s} \cdot \frac{1}{s} = \frac{1}{s \ln s}$$

$$29. f(x) = \ln(x^2 - 2x) \Rightarrow f'(x) = \frac{1}{x^2 - 2x} (2x - 2) = \frac{2(x - 1)}{x(x - 2)}.$$

$$\text{Dom}(f) = \{x \mid x(x - 2) > 0\} = (-\infty, 0) \cup (2, \infty).$$

$$41. y = \sqrt{\frac{x-1}{x^4+1}} \Rightarrow \ln y = \ln \left(\frac{x-1}{x^4+1} \right)^{1/2} \Rightarrow \ln y = \frac{1}{2} \ln(x-1) - \frac{1}{2} \ln(x^4+1) \Rightarrow$$

$$\frac{1}{y} y' = \frac{1}{2} \frac{1}{x-1} - \frac{1}{2} \frac{1}{x^4+1} \cdot 4x^3 \Rightarrow y' = y \left(\frac{1}{2(x-1)} - \frac{2x^3}{x^4+1} \right) \Rightarrow y' = \sqrt{\frac{x-1}{x^4+1}} \left(\frac{1}{2(x-1)} - \frac{2x^3}{x^4+1} \right)$$

$$48. y = (\sin x)^{\ln x} \Rightarrow \ln y = \ln(\sin x)^{\ln x} \Rightarrow \ln y = \ln x \cdot \ln \sin x \Rightarrow \frac{1}{y} y' = \ln x \cdot \frac{1}{\sin x} \cdot \cos x + \ln \sin x \cdot \frac{1}{x} \Rightarrow$$

$$y' = y \left(\ln x \cdot \frac{\cos x}{\sin x} + \frac{\ln \sin x}{x} \right) \Rightarrow y' = (\sin x)^{\ln x} \left(\ln x \cot x + \frac{\ln \sin x}{x} \right)$$