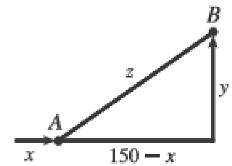
5. $V = \pi r^2 h = \pi(5)^2 h = 25\pi h \quad \Rightarrow \quad \dfrac{dV}{dt} = 25\pi \dfrac{dh}{dt} \quad \Rightarrow \quad 3 = 25\pi \dfrac{dh}{dt} \quad \Rightarrow \quad \dfrac{dh}{dt} = \dfrac{3}{25\pi}$ m/min.

14. (a) Given: at noon, ship A is 150 km west of ship B; ship A is sailing east at 35 km/h, and ship B is sailing north at 25 km/h. If we let t be time (in hours), x be the distance traveled by ship A (in km), and y be the distance traveled by ship B (in km), then we are given that $dx/dt = 35$ km/h and $dy/dt = 25$ km/h.

(b) Unknown: the rate at which the distance between the ships is changing at 4:00 PM. If we let z be the distance between the ships, then we want to find dz/dt when $t = 4$ h.

(c)

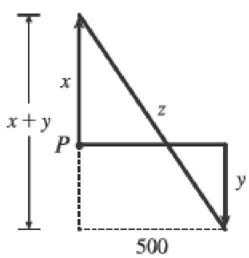


(d) $z^2 = (150 - x)^2 + y^2 \quad \Rightarrow \quad 2z \dfrac{dz}{dt} = 2(150 - x)\left(-\dfrac{dx}{dt}\right) + 2y \dfrac{dy}{dt}$

(e) At 4:00 PM, $x = 4(35) = 140$ and $y = 4(25) = 100 \quad \Rightarrow \quad z = \sqrt{(150 - 140)^2 + 100^2} = \sqrt{10{,}100}$.

So $\dfrac{dz}{dt} = \dfrac{1}{z}\left[(x - 150)\dfrac{dx}{dt} + y\dfrac{dy}{dt}\right] = \dfrac{-10(35) + 100(25)}{\sqrt{10{,}100}} = \dfrac{215}{\sqrt{101}} \approx 21.4$ km/h.

17.



We are given that $\dfrac{dx}{dt} = 4$ ft/s and $\dfrac{dy}{dt} = 5$ ft/s. $z^2 = (x + y)^2 + 500^2 \quad \Rightarrow$

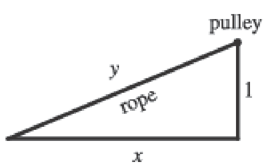
$2z\dfrac{dz}{dt} = 2(x + y)\left(\dfrac{dx}{dt} + \dfrac{dy}{dt}\right)$. 15 minutes after the woman starts, we have

$x = (4 \text{ ft/s})(20 \text{ min})(60 \text{ s/min}) = 4800$ ft and $y = 5 \cdot 15 \cdot 60 = 4500 \quad \Rightarrow$

$z = \sqrt{(4800 + 4500)^2 + 500^2} = \sqrt{86{,}740{,}000}$, so

$\dfrac{dz}{dt} = \dfrac{x + y}{z}\left(\dfrac{dx}{dt} + \dfrac{dy}{dt}\right) = \dfrac{4800 + 4500}{\sqrt{86{,}740{,}000}}(4 + 5) = \dfrac{837}{\sqrt{8674}} \approx 8.99$ ft/s.

20.



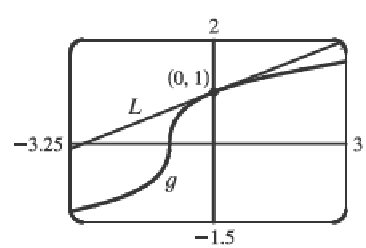
Given $\dfrac{dy}{dt} = -1$ m/s, find $\dfrac{dx}{dt}$ when $x = 8$ m. $y^2 = x^2 + 1 \quad \Rightarrow \quad 2y\dfrac{dy}{dt} = 2x\dfrac{dx}{dt} \quad \Rightarrow$

$\dfrac{dx}{dt} = \dfrac{y}{x}\dfrac{dy}{dt} = -\dfrac{y}{x}$. When $x = 8$, $y = \sqrt{65}$, so $\dfrac{dx}{dt} = -\dfrac{\sqrt{65}}{8}$. Thus, the boat approaches the dock at $\dfrac{\sqrt{65}}{8} \approx 1.01$ m/s.

6. $g(x) = \sqrt[3]{1 + x} = (1 + x)^{1/3} \quad \Rightarrow \quad g'(x) = \frac{1}{3}(1 + x)^{-2/3}$, so $g(0) = 1$ and $g'(0) = \frac{1}{3}$. Therefore, $\sqrt[3]{1 + x} = g(x) \approx g(0) + g'(0)(x - 0) = 1 + \frac{1}{3}x$.

So $\sqrt[3]{0.95} = \sqrt[3]{1 + (-0.05)} \approx 1 + \frac{1}{3}(-0.05) = 0.98\overline{3}$,

and $\sqrt[3]{1.1} = \sqrt[3]{1 + 0.1} \approx 1 + \frac{1}{3}(0.1) = 1.0\overline{3}$.



24. To estimate $e^{-0.015}$, we'll find the linearization of $f(x) = e^x$ at $a = 0$. Since $f'(x) = e^x$, $f(0) = 1$, and $f'(0) = 1$, we have $L(x) = 1 + 1(x - 0) = x + 1$. Thus, $e^x \approx x + 1$ when x is near 0, so $e^{-0.015} \approx -0.015 + 1 = 0.985$.

44. (a) $g'(x) = \sqrt{x^2+5} \quad \Rightarrow \quad g'(2) = \sqrt{9} = 3$. $g(1.95) \approx g(2) + g'(2)(1.95-2) = -4 + 3(-0.05) = -4.15$.

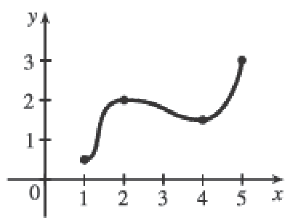
$g(2.05) \approx g(2) + g'(2)(2.05-2) = -4 + 3(0.05) = -3.85$.

(b) The formula $g'(x) = \sqrt{x^2+5}$ shows that $g'(x)$ is positive and increasing. This means that the slopes of the tangent lines are positive and the tangents are getting steeper. So the tangent lines lie *below* the graph of g. Hence, the estimates in part (a) are too small.

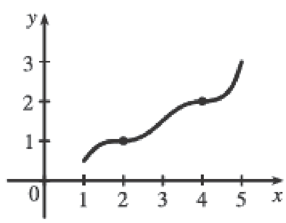
5. Absolute maximum value is $f(4) = 5$; there is no absolute minimum value; local maximum values are $f(4) = 5$ and $f(6) = 4$; local minimum values are $f(2) = 2$ and $f(1) = f(5) = 3$.

6. There is no absolute maximum value; absolute minimum value is $g(4) = 1$; local maximum values are $g(3) = 4$ and $g(6) = 3$; local minimum values are $g(2) = 2$ and $g(4) = 1$.

8. Absolute minimum at 1, absolute maximum at 5, local maximum at 2, local minimum at 4



10. f has no local maximum or minimum, but 2 and 4 are critical numbers



34. $g(t) = |3t-4| = \begin{cases} 3t-4 & \text{if } 3t-4 \geq 0 \\ -(3t-4) & \text{if } 3t-4 < 0 \end{cases} = \begin{cases} 3t-4 & \text{if } t \geq \frac{4}{3} \\ 4-3t & \text{if } t < \frac{4}{3} \end{cases}$

$g'(t) = \begin{cases} 3 & \text{if } t > \frac{4}{3} \\ -3 & \text{if } t < \frac{4}{3} \end{cases}$ and $g'(t)$ does not exist at $t = \frac{4}{3}$, so $t = \frac{4}{3}$ is a critical number.

36. $h(p) = \dfrac{p-1}{p^2+4} \quad \Rightarrow \quad h'(p) = \dfrac{(p^2+4)(1)-(p-1)(2p)}{(p^2+4)^2} = \dfrac{p^2+4-2p^2+2p}{(p^2+4)^2} = \dfrac{-p^2+2p+4}{(p^2+4)^2}$.

$h'(p) = 0 \quad \Rightarrow \quad p = \dfrac{-2 \pm \sqrt{4+16}}{-2} = 1 \pm \sqrt{5}$. The critical numbers are $1 \pm \sqrt{5}$. [$h'(p)$ exists for all real numbers.]

60. $f(x) = x - \ln x$, $[\frac{1}{2}, 2]$. $f'(x) = 1 - \frac{1}{x} = \frac{x-1}{x}$. $f'(x) = 0 \Rightarrow x = 1$. [Note that 0 is not in the domain of f.]

$f\left(\frac{1}{2}\right) = \frac{1}{2} - \ln\frac{1}{2} \approx 1.19$, $f(1) = 1$, and $f(2) = 2 - \ln 2 \approx 1.31$. So $f(2) = 2 - \ln 2$ is the absolute maximum value and $f(1) = 1$ is the absolute minimum value.