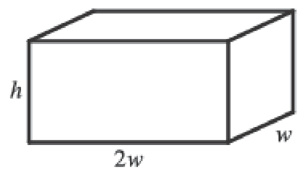
2. The two numbers are $x+100$ and x. Minimize $f(x)=(x+100)x=x^2+100x$. $f'(x)=2x+100=0 \;\Rightarrow\; x=-50$. Since $f''(x)=2>0$, there is an absolute minimum at $x=-50$. The two numbers are 50 and -50.

16.



$V=lwh \;\Rightarrow\; 10=(2w)(w)h=2w^2h$, so $h=5/w^2$.

The cost is $10(2w^2)+6[2(2wh)+2(hw)]=20w^2+36wh$, so

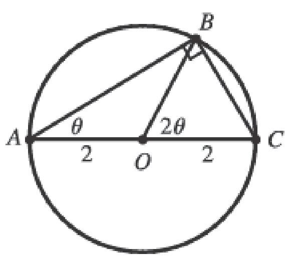
$C(w)=20w^2+36w\left(5/w^2\right)=20w^2+180/w$.

$C'(w)=40w-180/w^2=40\left(w^3-\frac{9}{2}\right)/w^2 \;\Rightarrow\; w=\sqrt[3]{\frac{9}{2}}$ is the critical number. There is an absolute minimum for C when $w=\sqrt[3]{\frac{9}{2}}$ since $C'(w)<0$ for $0<w<\sqrt[3]{\frac{9}{2}}$ and $C'(w)>0$ for $w>\sqrt[3]{\frac{9}{2}}$.

$C\left(\sqrt[3]{\frac{9}{2}}\right)=20\left(\sqrt[3]{\frac{9}{2}}\right)^2+\dfrac{180}{\sqrt[3]{9/2}}\approx \$163.54$.

20. The distance d from the point $(3,0)$ to a point $(x,\sqrt{x})$ on the curve is given by $d=\sqrt{(x-3)^2+(\sqrt{x}-0)^2}$ and the square of the distance is $S=d^2=(x-3)^2+x$. $S'=2(x-3)+1=2x-5$ and $S'=0 \;\Leftrightarrow\; x=\frac{5}{2}$. Now $S''=2>0$, so we know that S has a minimum at $x=\frac{5}{2}$. Thus, the y-value is $\sqrt{\frac{5}{2}}$ and the point is $\left(\frac{5}{2},\sqrt{\frac{5}{2}}\right)$.

48.



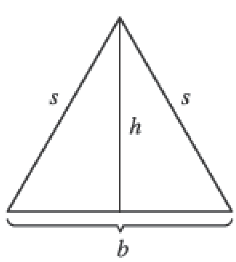
In isosceles triangle AOB, $\angle O=180^\circ-\theta-\theta$, so $\angle BOC=2\theta$. The distance rowed is $4\cos\theta$ while the distance walked is the length of arc $BC=2(2\theta)=4\theta$. The time taken is given by $T(\theta)=\dfrac{4\cos\theta}{2}+\dfrac{4\theta}{4}=2\cos\theta+\theta,\;0\le\theta\le\frac{\pi}{2}$.

$T'(\theta)=-2\sin\theta+1=0 \;\Leftrightarrow\; \sin\theta=\frac{1}{2} \;\Rightarrow\; \theta=\frac{\pi}{6}$.

Check the value of T at $\theta=\frac{\pi}{6}$ and at the endpoints of the domain of T; that is, $\theta=0$ and $\theta=\frac{\pi}{2}$.

$T(0)=2$, $T\left(\frac{\pi}{6}\right)=\sqrt{3}+\frac{\pi}{6}\approx 2.26$, and $T\left(\frac{\pi}{2}\right)=\frac{\pi}{2}\approx 1.57$. Therefore, the minimum value of T is $\frac{\pi}{2}$ when $\theta=\frac{\pi}{2}$; that is, the woman should walk all the way. Note that $T''(\theta)=-2\cos\theta<0$ for $0\le\theta<\frac{\pi}{2}$, so $\theta=\frac{\pi}{6}$ gives a maximum time.

63.



Here $s^2=h^2+b^2/4$, so $h^2=s^2-b^2/4$. The area is $A=\frac{1}{2}b\sqrt{s^2-b^2/4}$.

Let the perimeter be p, so $2s+b=p$ or $s=(p-b)/2 \;\Rightarrow$

$A(b)=\frac{1}{2}b\sqrt{(p-b)^2/4-b^2/4}=b\sqrt{p^2-2pb}/4$. Now

$A'(b)=\dfrac{\sqrt{p^2-2pb}}{4}-\dfrac{bp/4}{\sqrt{p^2-2pb}}=\dfrac{-3pb+p^2}{4\sqrt{p^2-2pb}}$.

Therefore, $A'(b)=0 \;\Rightarrow\; -3pb+p^2=0 \;\Rightarrow\; b=p/3$. Since $A'(b)>0$ for $b<p/3$ and $A'(b)<0$ for $b>p/3$, there is an absolute maximum when $b=p/3$. But then $2s+p/3=p$, so $s=p/3 \;\Rightarrow\; s=b \;\Rightarrow$ the triangle is equilateral.

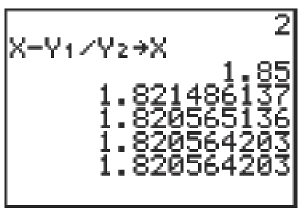
7. $f(x) = x^5 - x - 1 \quad \Rightarrow \quad f'(x) = 5x^4 - 1$, so $x_{n+1} = x_n - \dfrac{x_n^5 - x_n - 1}{5x_n^4 - 1}$. Now $x_1 = 1 \quad \Rightarrow$

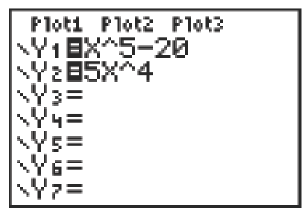
$x_2 = 1 - \dfrac{1 - 1 - 1}{5 - 1} = 1 - \left(-\frac{1}{4}\right) = 1.25 \quad \Rightarrow \quad x_3 = 1.25 - \dfrac{(1.25)^5 - 1.25 - 1}{5(1.25)^4 - 1} \approx 1.1785.$

11. To approximate $x = \sqrt[5]{20}$ (so that $x^5 = 20$), we can take $f(x) = x^5 - 20$. So $f'(x) = 5x^4$, and thus,

$x_{n+1} = x_n - \dfrac{x_n^5 - 20}{5x_n^4}$. Since $\sqrt[5]{32} = 2$ and 32 is reasonably close to 20, we'll use $x_1 = 2$. We need to find approximations until they agree to eight decimal places. $x_1 = 2 \quad \Rightarrow \quad x_2 = 1.85$, $x_3 \approx 1.82148614$, $x_4 \approx 1.82056514$, $x_5 \approx 1.82056420 \approx x_6$. So $\sqrt[5]{20} \approx 1.82056420$, to eight decimal places.

Here is a quick and easy method for finding the iterations for Newton's method on a programmable calculator. (The screens shown are from the TI-84 Plus, but the method is similar on other calculators.) Assign $f(x) = x^5 - 20$ to Y_1, and $f'(x) = 5x^4$ to Y_2. Now store $x_1 = 2$ in X and then enter $\mathrm{X} - \mathrm{Y}_1/\mathrm{Y}_2 \rightarrow \mathrm{X}$ to get $x_2 = 1.85$. By successively pressing the ENTER key, you get the approximations $x_3, x_4, \ldots$.



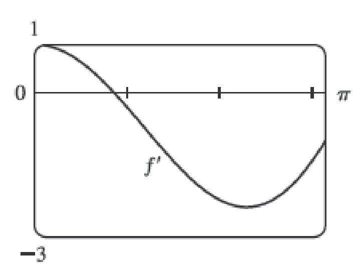


In Derive, load the utility file `SOLVE`. Enter `NEWTON(x^5-20,x,2)` and then `APPROXIMATE` to get [2, 1.85, 1.82148614, 1.82056514, 1.82056420]. You can request a specific iteration by adding a fourth argument. For example, `NEWTON(x^5-20,x,2,2)` gives [2, 1.85, 1.82148614].

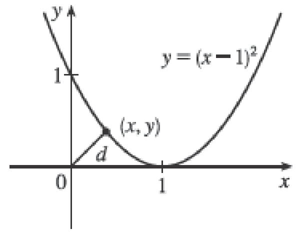
In Maple, make the assignments $f := x \rightarrow x\hat{\ }5 - 20$;, $g := x \rightarrow x - f(x)/D(f)(x)$;, and $x := 2.$;. Repeatedly execute the command $x := g(x)$; to generate successive approximations.

In Mathematica, make the assignments $f[x\_] := x\hat{\ }5 - 20$, $g[x\_] := x - f[x]/f'[x]$, and $x = 2$. Repeatedly execute the command $x = g[x]$ to generate successive approximations.

36. $f(x) = x\cos x \quad \Rightarrow \quad f'(x) = \cos x - x\sin x$. $f'(x)$ exists for all x, so to find the maximum of f, we can examine the zeros of f'. From the graph of f', we see that a good choice for x_1 is $x_1 = 0.9$. Use $g(x) = \cos x - x\sin x$ and $g'(x) = -2\sin x - x\cos x$ to obtain $x_2 \approx 0.860781$, $x_3 \approx 0.860334 \approx x_4$. Now we have $f(0) = 0$, $f(\pi) = -\pi$, and $f(0.860334) \approx 0.561096$, so 0.561096 is the absolute maximum value of f correct to six decimal places.



39. We need to minimize the distance from $(0, 0)$ to an arbitrary point (x, y) on the curve $y = (x-1)^2$. $d = \sqrt{x^2 + y^2} \quad \Rightarrow$



$d(x) = \sqrt{x^2 + [(x-1)^2]^2} = \sqrt{x^2 + (x-1)^4}$. When $d' = 0$, d will be minimized and equivalently, $s = d^2$ will be minimized, so we will use Newton's method with $f = s'$ and $f' = s''$.

$f(x) = 2x + 4(x-1)^3 \quad \Rightarrow \quad f'(x) = 2 + 12(x-1)^2$, so $x_{n+1} = x_n - \dfrac{2x_n + 4(x_n - 1)^3}{2 + 12(x_n - 1)^2}$. Try $x_1 = 0.5 \quad \Rightarrow$ $x_2 = 0.4$, $x_3 \approx 0.410127$, $x_4 \approx 0.410245 \approx x_5$. Now $d(0.410245) \approx 0.537841$ is the minimum distance and the point on the parabola is $(0.410245, 0.347810)$, correct to six decimal places.