

26. $f''(x) = x^6 - 4x^4 + x + 1 \Rightarrow f'(x) = \frac{1}{7}x^7 - \frac{4}{5}x^5 + \frac{1}{2}x^2 + x + C \Rightarrow$

$$f(x) = \frac{1}{56}x^8 - \frac{2}{15}x^6 + \frac{1}{6}x^3 + \frac{1}{2}x^2 + Cx + D$$

42. $f''(t) = 3/\sqrt{t} = 3t^{-1/2} \Rightarrow f'(t) = 6t^{1/2} + C$. $f'(4) = 12 + C$ and $f'(4) = 7 \Rightarrow C = -5$, so $f'(t) = 6t^{1/2} - 5$

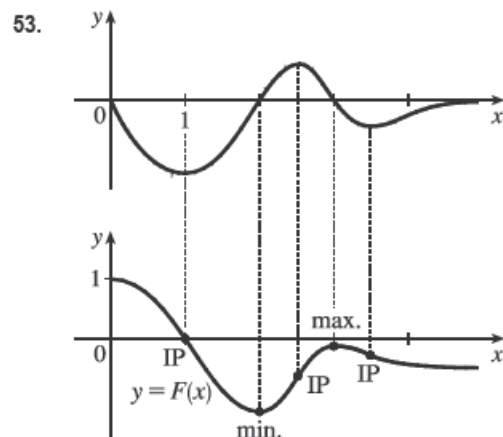
and hence, $f(t) = 4t^{3/2} - 5t + D$. $f(4) = 32 - 20 + D$ and $f(4) = 20 \Rightarrow D = 8$, so $f(t) = 4t^{3/2} - 5t + 8$.

48. $f'''(x) = \cos x \Rightarrow f''(x) = \sin x + C$. $f''(0) = C$ and $f''(0) = 3 \Rightarrow C = 3$. $f''(x) = \sin x + 3 \Rightarrow$

$$f'(x) = -\cos x + 3x + D$$
. $f'(0) = -1 + D$ and $f'(0) = 2 \Rightarrow D = 3$. $f'(x) = -\cos x + 3x + 3 \Rightarrow$

$$f(x) = -\sin x + \frac{3}{2}x^2 + 3x + E$$
. $f(0) = E$ and $f(0) = 1 \Rightarrow E = 1$. Thus, $f(x) = -\sin x + \frac{3}{2}x^2 + 3x + 1$.

52. We know right away that c cannot be f 's antiderivative, since the slope of c is not zero at the x -value where $f = 0$. Now f is positive when a is increasing and negative when a is decreasing, so a is the antiderivative of f .



The graph of F must start at $(0, 1)$. Where the given graph, $y = f(x)$, has a

local minimum or maximum, the graph of F will have an inflection point.

Where f is negative (positive), F is decreasing (increasing).

Where f changes from negative to positive, F will have a minimum.

Where f changes from positive to negative, F will have a maximum.

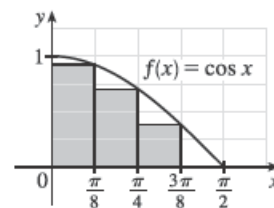
Where f is decreasing (increasing), F is concave downward (upward).

74. $v'(t) = a(t) = -22$. The initial velocity is $50 \text{ mi/h} = \frac{50 \cdot 5280}{3600} = \frac{220}{3} \text{ ft/s}$, so $v(t) = -22t + \frac{220}{3}$.

The car stops when $v(t) = 0 \Leftrightarrow t = \frac{220}{3 \cdot 22} = \frac{10}{3}$. Since $s(t) = -11t^2 + \frac{220}{3}t$, the distance covered is

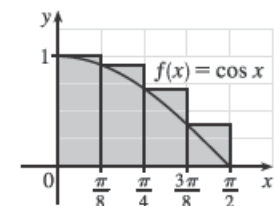
$$s\left(\frac{10}{3}\right) = -11\left(\frac{10}{3}\right)^2 + \frac{220}{3} \cdot \frac{10}{3} = \frac{1100}{9} = 122.\bar{2} \text{ ft.}$$

$$\begin{aligned}
 3. (a) R_4 &= \sum_{i=1}^4 f(x_i) \Delta x \quad \left[\Delta x = \frac{\pi/2 - 0}{4} = \frac{\pi}{8} \right] = \left[\sum_{i=1}^4 f(x_i) \right] \Delta x \\
 &= [f(x_1) + f(x_2) + f(x_3) + f(x_4)] \Delta x \\
 &= \left[\cos \frac{\pi}{8} + \cos \frac{2\pi}{8} + \cos \frac{3\pi}{8} + \cos \frac{4\pi}{8} \right] \frac{\pi}{8} \\
 &\approx (0.9239 + 0.7071 + 0.3827 + 0) \frac{\pi}{8} \approx 0.7908
 \end{aligned}$$



Since f is *decreasing* on $[0, \pi/2]$, an *underestimate* is obtained by using the *right* endpoint approximation, R_4 .

$$\begin{aligned}
 (b) L_4 &= \sum_{i=1}^4 f(x_{i-1}) \Delta x = \left[\sum_{i=1}^4 f(x_{i-1}) \right] \Delta x \\
 &= [f(x_0) + f(x_1) + f(x_2) + f(x_3)] \Delta x \\
 &= \left[\cos 0 + \cos \frac{\pi}{8} + \cos \frac{2\pi}{8} + \cos \frac{3\pi}{8} \right] \frac{\pi}{8} \\
 &\approx (1 + 0.9239 + 0.7071 + 0.3827) \frac{\pi}{8} \approx 1.1835
 \end{aligned}$$



L_4 is an overestimate. Alternatively, we could just add the area of the leftmost upper rectangle and subtract the area of the rightmost lower rectangle; that is, $L_4 = R_4 + f(0) \cdot \frac{\pi}{8} - f(\frac{\pi}{2}) \cdot \frac{\pi}{8}$.

18. For an increasing function, using left endpoints gives us an underestimate and using right endpoints results in an overestimate.

We will use M_6 to get an estimate. $\Delta t = \frac{30-0}{6} = 5 \text{ s} = \frac{5}{3600} \text{ h} = \frac{1}{720} \text{ h}$.

$$\begin{aligned}
 M_6 &= \frac{1}{720} [v(2.5) + v(7.5) + v(12.5) + v(17.5) + v(22.5) + v(27.5)] \\
 &= \frac{1}{720} (31.25 + 66 + 88 + 103.5 + 113.75 + 119.25) = \frac{1}{720} (521.75) \approx 0.725 \text{ km}
 \end{aligned}$$

For a very rough check on the above calculation, we can draw a line from $(0, 0)$ to $(30, 120)$ and calculate the area of the triangle: $\frac{1}{2}(30)(120) = 1800$. Divide by 3600 to get 0.5, which is clearly an underestimate, making our midpoint estimate of 0.725 seem reasonable. Of course, answers will vary due to different readings of the graph.

22. $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{2}{n} \left(5 + \frac{2i}{n} \right)^{10}$ can be interpreted as the area of the region lying under the graph of $y = (5 + x)^{10}$ on the interval

$[0, 2]$, since for $y = (5 + x)^{10}$ on $[0, 2]$ with $\Delta x = \frac{2-0}{n} = \frac{2}{n}$, $x_i = 0 + i \Delta x = \frac{2i}{n}$, and $x_i^* = x_i$, the expression for the

area is $A = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*) \Delta x = \lim_{n \rightarrow \infty} \sum_{i=1}^n \left(5 + \frac{2i}{n} \right)^{10} \frac{2}{n}$. Note that the answer is not unique. We could use $y = x^{10}$ on $[5, 7]$ or, in general, $y = ((5 - n) + x)^{10}$ on $[n, n + 2]$.

17. On $[2, 6]$, $\lim_{n \rightarrow \infty} \sum_{i=1}^n x_i \ln(1 + x_i^2) \Delta x = \int_2^6 x \ln(1 + x^2) dx$.

18. On $[\pi, 2\pi]$, $\lim_{n \rightarrow \infty} \sum_{i=1}^n \frac{\cos x_i}{x_i} \Delta x = \int_{\pi}^{2\pi} \frac{\cos x}{x} dx$.

34. (a) $\int_0^2 g(x) dx = \frac{1}{2} \cdot 4 \cdot 2 = 4$ [area of a triangle]

(b) $\int_2^6 g(x) dx = -\frac{1}{2}\pi(2)^2 = -2\pi$ [negative of the area of a semicircle]

(c) $\int_6^7 g(x) dx = \frac{1}{2} \cdot 1 \cdot 1 = \frac{1}{2}$ [area of a triangle]

$$\int_0^7 g(x) dx = \int_0^2 g(x) dx + \int_2^6 g(x) dx + \int_6^7 g(x) dx = 4 - 2\pi + \frac{1}{2} = 4.5 - 2\pi$$

48. $\int_1^4 f(x) dx = \int_1^5 f(x) dx - \int_4^5 f(x) dx = 12 - 3.6 = 8.4$

50. If $f(x) = \begin{cases} 3 & \text{for } x < 3 \\ x & \text{for } x \geq 3 \end{cases}$, then $\int_0^5 f(x) dx$ can be interpreted as the area of the shaded

region, which consists of a 5-by-3 rectangle surmounted by an isosceles right triangle

whose legs have length 2. Thus, $\int_0^5 f(x) dx = 5(3) + \frac{1}{2}(2)(2) = 17$.

