29. $y^2 + 2y = 4x^2 + 3 \quad \Leftrightarrow \quad y^2 + 2y + 1 = 4x^2 + 4 \quad \Leftrightarrow \quad (y+1)^2 - 4x^2 = 4 \quad \Leftrightarrow \quad \dfrac{(y+1)^2}{4} - x^2 = 1$. This is an equation of a *hyperbola* with vertices $(0, -1 \pm 2) = (0, 1)$ and $(0, -3)$. The foci are at $\left(0, -1 \pm \sqrt{4+1}\right) = \left(0, -1 \pm \sqrt{5}\right)$.

42. Foci $F_1(-4, 0)$ and $F_2(4, 0) \quad \Rightarrow \quad c = 4$ and an equation is $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$. The ellipse passes through $P(-4, 1.8)$, so

$2a = |PF_1| + |PF_2| \quad \Rightarrow \quad 2a = 1.8 + \sqrt{8^2 + (1.8)^2} \quad \Rightarrow \quad 2a = 1.8 + 8.2 \quad \Rightarrow \quad a = 5.$

$b^2 = a^2 - c^2 = 25 - 16 = 9$ and the equation is $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$.

47. The center of a hyperbola with vertices $(\pm 3, 0)$ is $(0, 0)$, so $a = 3$ and an equation is $\dfrac{x^2}{3^2} - \dfrac{y^2}{b^2} = 1$.

Asymptotes $y = \pm 2x \quad \Rightarrow \quad \dfrac{b}{a} = 2 \quad \Rightarrow \quad b = 2(3) = 6$ and the equation is $\dfrac{x^2}{9} - \dfrac{y^2}{36} = 1$.

54. We can follow exactly the same sequence of steps as in the derivation of Formula 4, except we use the points $(1, 1)$ and $(-1, -1)$ in the distance formula (first equation of that derivation) so $\sqrt{(x-1)^2 + (y-1)^2} + \sqrt{(x+1)^2 + (y+1)^2} = 4$ will lead (after moving the second term to the right, squaring, and simplifying) to $2\sqrt{(x+1)^2 + (y+1)^2} = x + y + 4$, which, after squaring and simplifying again, leads to $3x^2 - 2xy + 3y^2 = 8$.

11. An equation of the sphere with center $(-3, 2, 5)$ and radius 4 is $[x - (-3)]^2 + (y-2)^2 + (z-5)^2 = 4^2$ or $(x+3)^2 + (y-2)^2 + (z-5)^2 = 16$. The intersection of this sphere with the yz-plane is the set of points on the sphere whose x-coordinate is 0. Putting $x = 0$ into the equation, we have $9 + (y-2)^2 + (z-5)^2 = 16$, $x = 0$ or $(y-2)^2 + (z-5)^2 = 7$, $x = 0$, which represents a circle in the yz-plane with center $(0, 2, 5)$ and radius $\sqrt{7}$.

17. Completing squares in the equation $2x^2 - 8x + 2y^2 + 2z^2 + 24z = 1$ gives

$2(x^2 - 4x + 4) + 2y^2 + 2(z^2 + 12z + 36) = 1 + 8 + 72 \quad \Rightarrow \quad 2(x-2)^2 + 2y^2 + 2(z+6)^2 = 81 \quad \Rightarrow$

$(x-2)^2 + y^2 + (z+6)^2 = \frac{81}{2}$, which we recognize as an equation of a sphere with center $(2, 0, -6)$ and radius $\sqrt{\frac{81}{2}} = 9/\sqrt{2}$.

40.
Let $P = (x, y, z)$. Then $2\,|PB| = |PA| \quad \Leftrightarrow \quad 4\,|PB|^2 = |PA|^2 \quad \Leftrightarrow$

$4\big((x-6)^2 + (y-2)^2 + (z+2)^2\big) = (x+1)^2 + (y-5)^2 + (z-3)^2 \quad \Leftrightarrow$

$4\big(x^2 - 12x + 36\big) - x^2 - 2x + 4\big(y^2 - 4y + 4\big) - y^2 + 10y + 4\big(z^2 + 4z + 4\big) - z^2 + 6z = 35 \quad \Leftrightarrow$

$3x^2 - 50x + 3y^2 - 6y + 3z^2 + 22z = 35 - 144 - 16 - 16 \quad \Leftrightarrow \quad x^2 - \frac{50}{3}x + y^2 - 2y + z^2 + \frac{22}{3}z = -\frac{141}{3}$.

By completing the square three times we get $\left(x - \frac{25}{3}\right)^2 + (y-1)^2 + \left(z + \frac{11}{3}\right)^2 = \frac{332}{9}$, which is an equation of a sphere with center $\left(\frac{25}{3}, 1, -\frac{11}{3}\right)$ and radius $\frac{\sqrt{332}}{3}$.

41.
We need to find a set of points $\big\{P(x, y, z) \;\big|\; |AP| = |BP|\big\}$.

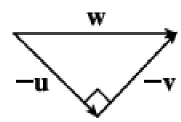
$\sqrt{(x+1)^2 + (y-5)^2 + (z-3)^2} = \sqrt{(x-6)^2 + (y-2)^2 + (z+2)^2} \quad \Rightarrow$

$(x+1)^2 + (y-5) + (z-3)^2 = (x-6)^2 + (y-2)^2 + (z+2)^2 \quad \Rightarrow$

$x^2 + 2x + 1 + y^2 - 10y + 25 + z^2 - 6z + 9 = x^2 - 12x + 36 + y^2 - 4y + 4 + z^2 + 4z + 4 \quad \Rightarrow \quad 14x - 6y - 10z = 9$.

Thus the set of points is a plane perpendicular to the line segment joining A and B (since this plane must contain the perpendicular bisector of the line segment AB).

8. We are given $\mathbf{u} + \mathbf{v} + \mathbf{w} = \mathbf{0}$, so $\mathbf{w} = (-\mathbf{u}) + (-\mathbf{v})$. (See the figure.)



Vectors $-\mathbf{u}$, $-\mathbf{v}$, and $\mathbf{w}$ form a right triangle, so from the Pythagorean Theorem we have $|-\mathbf{u}|^2 + |-\mathbf{v}|^2 = |\mathbf{w}|^2$. But $|-\mathbf{u}| = |\mathbf{u}| = 1$ and $|-\mathbf{v}| = |\mathbf{v}| = 1$ so $|\mathbf{w}| = \sqrt{|-\mathbf{u}|^2 + |-\mathbf{v}|^2} = \sqrt{2}$.

22. $\mathbf{a} + \mathbf{b} = (2\,\mathbf{i} - 4\,\mathbf{j} + 4\,\mathbf{k}) + (2\,\mathbf{j} - \mathbf{k}) = 2\,\mathbf{i} - 2\,\mathbf{j} + 3\,\mathbf{k}$

$2\mathbf{a} + 3\mathbf{b} = 2\,(2\,\mathbf{i} - 4\,\mathbf{j} + 4\,\mathbf{k}) + 3\,(2\,\mathbf{j} - \mathbf{k}) = 4\,\mathbf{i} - 8\,\mathbf{j} + 8\,\mathbf{k} + 6\,\mathbf{j} - 3\,\mathbf{k} = 4\,\mathbf{i} - 2\,\mathbf{j} + 5\,\mathbf{k}$

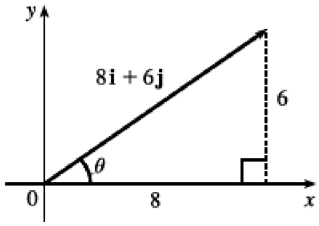
$|\mathbf{a}| = \sqrt{2^2 + (-4)^2 + 4^2} = \sqrt{36} = 6$

$|\mathbf{a} - \mathbf{b}| = |(2\,\mathbf{i} - 4\,\mathbf{j} + 4\,\mathbf{k}) - (2\,\mathbf{j} - \mathbf{k})| = |2\,\mathbf{i} - 6\,\mathbf{j} + 5\,\mathbf{k}| = \sqrt{2^2 + (-6)^2 + 5^2} = \sqrt{65}$

26. $|\langle -2, 4, 2\rangle| = \sqrt{(-2)^2 + 4^2 + 2^2} = \sqrt{24} = 2\sqrt{6}$, so a unit vector in the direction of $\langle -2, 4, 2\rangle$ is $\mathbf{u} = \dfrac{1}{2\sqrt{6}}\,\langle -2, 4, 2\rangle$.

A vector in the same direction but with length 6 is $6\mathbf{u} = 6 \cdot \dfrac{1}{2\sqrt{6}}\,\langle -2, 4, 2\rangle = \left\langle -\dfrac{6}{\sqrt{6}}, \dfrac{12}{\sqrt{6}}, \dfrac{6}{\sqrt{6}} \right\rangle$ or $\langle -\sqrt{6}, 2\sqrt{6}, \sqrt{6}\,\rangle$.

28.



From the figure we see that $\tan\theta = \frac{6}{8} = \frac{3}{4}$, so $\theta = \tan^{-1}\left(\frac{3}{4}\right) \approx 36.9°$.

35. With respect to the water's surface, the woman's velocity is the vector sum of the velocity of the ship with respect to the water, and the woman's velocity with respect to the ship. If we let north be the positive y-direction, then $\mathbf{v} = \langle 0, 22\rangle + \langle -3, 0\rangle = \langle -3, 22\rangle$. The woman's speed is $|\mathbf{v}| = \sqrt{9+484} \approx 22.2$ mi/h. The vector $\mathbf{v}$ makes an angle θ with the east, where $\theta = \tan^{-1}\left(\frac{22}{-3}\right) \approx 98°$. Therefore, the woman's direction is about $\mathrm{N}(98-90)°\mathrm{W} = \mathrm{N}8°\mathrm{W}$.

43. By the Triangle Law, $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$. Then $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \overrightarrow{AC} + \overrightarrow{CA}$, but $\overrightarrow{AC} + \overrightarrow{CA} = \overrightarrow{AC} + \left(-\overrightarrow{AC}\right) = \mathbf{0}$.

So $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = \mathbf{0}$.

44. $\overrightarrow{AC} = \frac{1}{3}\overrightarrow{AB}$ and $\overrightarrow{BC} = \frac{2}{3}\overrightarrow{BA}$. $\mathbf{c} = \overrightarrow{OA} + \overrightarrow{AC} = \mathbf{a} + \frac{1}{3}\overrightarrow{AB} \quad\Rightarrow\quad \overrightarrow{AB} = 3\,\mathbf{c} - 3\,\mathbf{a}$. $\mathbf{c} = \overrightarrow{OB} + \overrightarrow{BC} = \overrightarrow{OA} + \frac{2}{3}\overrightarrow{BA} \quad\Rightarrow$ $\overrightarrow{BA} = \frac{3}{2}\,\mathbf{c} - \frac{3}{2}\,\mathbf{b}$. $\overrightarrow{BA} = -\overrightarrow{AB}$, so $\frac{3}{2}\,\mathbf{c} - \frac{3}{2}\,\mathbf{b} = 3\,\mathbf{a} - 3\,\mathbf{c} \quad\Leftrightarrow\quad \mathbf{c} + 2\,\mathbf{c} = 2\,\mathbf{a} + \mathbf{b} \quad\Leftrightarrow\quad \mathbf{c} = \frac{2}{3}\,\mathbf{a} + \frac{1}{3}\,\mathbf{b}$.

26. By Theorem 3, vectors $\langle 2, 1, -1\rangle$ and $\langle 1, x, 0\rangle$ meet at an angle of $45°$ when

$\langle 2,1,-1\rangle \cdot \langle 1,x,0\rangle = \sqrt{4+1+1}\,\sqrt{1+x^2+0}\,\cos 45°$ or $2 + x - 0 = \sqrt{6}\,\sqrt{1+x^2}\cdot\frac{\sqrt{2}}{2} \quad\Leftrightarrow\quad 2 + x = \sqrt{3}\sqrt{1+x^2}$.

Squaring both sides gives $4 + 4x + x^2 = 3 + 3x^2 \quad\Leftrightarrow\quad 2x^2 - 4x - 1 = 0$. By the quadratic formula,

$x = \dfrac{-(-4) \pm \sqrt{(-4)^2 - 4(2)(-1)}}{2(2)} = \dfrac{4 \pm \sqrt{24}}{4} = \dfrac{4 \pm 2\sqrt{6}}{4} = 1 \pm \dfrac{\sqrt{6}}{2}$. (You can verify that both values are valid.)

27.
Let $\mathbf{a} = a_1\,\mathbf{i} + a_2\,\mathbf{j} + a_3\,\mathbf{k}$ be a vector orthogonal to both $\mathbf{i}+\mathbf{j}$ and $\mathbf{i}+\mathbf{k}$. Then $\mathbf{a}\cdot(\mathbf{i}+\mathbf{j}) = 0 \quad\Leftrightarrow\quad a_1 + a_2 = 0$ and $\mathbf{a}\cdot(\mathbf{i}+\mathbf{k}) = 0 \quad\Leftrightarrow\quad a_1 + a_3 = 0$, so $a_1 = -a_2 = -a_3$. Furthermore $\mathbf{a}$ is to be a unit vector, so $1 = a_1^2 + a_2^2 + a_3^2 = 3a_1^2$ implies $a_1 = \pm\frac{1}{\sqrt{3}}$. Thus $\mathbf{a} = \frac{1}{\sqrt{3}}\,\mathbf{i} - \frac{1}{\sqrt{3}}\,\mathbf{j} - \frac{1}{\sqrt{3}}\,\mathbf{k}$ and $\mathbf{a} = -\frac{1}{\sqrt{3}}\,\mathbf{i} + \frac{1}{\sqrt{3}}\,\mathbf{j} + \frac{1}{\sqrt{3}}\,\mathbf{k}$ are two such unit vectors.

45. $(\mathrm{orth}_{\mathbf{a}}\,\mathbf{b})\cdot\mathbf{a} = (\mathbf{b} - \mathrm{proj}_{\mathbf{a}}\,\mathbf{b})\cdot\mathbf{a} = \mathbf{b}\cdot\mathbf{a} - (\mathrm{proj}_{\mathbf{a}}\,\mathbf{b})\cdot\mathbf{a} = \mathbf{b}\cdot\mathbf{a} - \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a}\cdot\mathbf{a} = \mathbf{b}\cdot\mathbf{a} - \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\,|\mathbf{a}|^2 = \mathbf{b}\cdot\mathbf{a} - \mathbf{a}\cdot\mathbf{b} = 0$.

So they are orthogonal by (7).

47. $\mathrm{comp}_{\mathbf{a}}\,\mathbf{b} = \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = 2 \quad\Leftrightarrow\quad \mathbf{a}\cdot\mathbf{b} = 2\,|\mathbf{a}| = 2\,\sqrt{10}$. If $\mathbf{b} = \langle b_1, b_2, b_3\rangle$, then we need $3b_1 + 0b_2 - 1b_3 = 2\,\sqrt{10}$.

One possible solution is obtained by taking $b_1 = 0$, $b_2 = 0$, $b_3 = -2\,\sqrt{10}$. In general, $\mathbf{b} = \left\langle s, t, 3s - 2\,\sqrt{10}\,\right\rangle$, $s, t \in \mathbb{R}$.

48. (a) $\text{comp}_{\mathbf{a}}\,\mathbf{b} = \text{comp}_{\mathbf{b}}\,\mathbf{a} \quad\Leftrightarrow\quad \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|} = \dfrac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{b}|} \quad\Leftrightarrow\quad \dfrac{1}{|\mathbf{a}|} = \dfrac{1}{|\mathbf{b}|}$ or $\mathbf{a}\cdot\mathbf{b} = 0 \quad\Leftrightarrow\quad |\mathbf{b}| = |\mathbf{a}|$ or $\mathbf{a}\cdot\mathbf{b} = 0$.

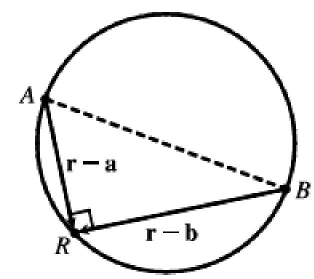
That is, if $\mathbf{a}$ and $\mathbf{b}$ are orthogonal or if they have the same length.

(b) $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \text{proj}_{\mathbf{b}}\,\mathbf{a} \quad\Leftrightarrow\quad \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a} = \dfrac{\mathbf{b}\cdot\mathbf{a}}{|\mathbf{b}|^2}\,\mathbf{b} \quad\Leftrightarrow\quad \mathbf{a}\cdot\mathbf{b} = 0 \quad \text{or} \quad \dfrac{\mathbf{a}}{|\mathbf{a}|^2} = \dfrac{\mathbf{b}}{|\mathbf{b}|^2}$.

But $\dfrac{\mathbf{a}}{|\mathbf{a}|^2} = \dfrac{\mathbf{b}}{|\mathbf{b}|^2} \quad\Rightarrow\quad \dfrac{|\mathbf{a}|}{|\mathbf{a}|^2} = \dfrac{|\mathbf{b}|}{|\mathbf{b}|^2} \quad\Rightarrow\quad |\mathbf{a}| = |\mathbf{b}|$. Substituting this into the previous equation gives $\mathbf{a} = \mathbf{b}$.

So $\text{proj}_{\mathbf{a}}\,\mathbf{b} = \text{proj}_{\mathbf{b}}\,\mathbf{a} \quad\Leftrightarrow\quad$ $\mathbf{a}$ and $\mathbf{b}$ are orthogonal, or they are equal.

54. $(\mathbf{r}-\mathbf{a})\cdot(\mathbf{r}-\mathbf{b}) = 0$ implies that the vectors $\mathbf{r}-\mathbf{a}$ and $\mathbf{r}-\mathbf{b}$ are orthogonal.



From the diagram (in which A, B and R are the terminal points of the vectors), we see that this implies that R lies on a sphere whose diameter is the line from A to B. The center of this circle is the midpoint of AB, that is,

$\frac{1}{2}(\mathbf{a}+\mathbf{b}) = \left\langle \frac{1}{2}(a_1+b_1), \frac{1}{2}(a_2+b_2), \frac{1}{2}(a_3+b_3)\right\rangle$, and its radius is

$\frac{1}{2}\,|\mathbf{a}-\mathbf{b}| = \frac{1}{2}\,\sqrt{(a_1-b_1)^2+(a_2-b_2)^2+(a_3-b_3)^2}$.

Or: Expand the given equation, substitute $\mathbf{r}\cdot\mathbf{r} = x^2+y^2+z^2$ and complete the squares.

64. If the vectors $\mathbf{u}+\mathbf{v}$ and $\mathbf{u}-\mathbf{v}$ are orthogonal then $(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) = 0$. But

$$\begin{aligned}
(\mathbf{u}+\mathbf{v})\cdot(\mathbf{u}-\mathbf{v}) &= (\mathbf{u}+\mathbf{v})\cdot\mathbf{u} - (\mathbf{u}+\mathbf{v})\cdot\mathbf{v} && \text{by Property 3 of the dot product}\\
&= \mathbf{u}\cdot\mathbf{u} + \mathbf{v}\cdot\mathbf{u} - \mathbf{u}\cdot\mathbf{v} - \mathbf{v}\cdot\mathbf{v} && \text{by Property 3}\\
&= |\mathbf{u}|^2 + \mathbf{u}\cdot\mathbf{v} - \mathbf{u}\cdot\mathbf{v} - |\mathbf{v}|^2 && \text{by Properties 1 and 2}\\
&= |\mathbf{u}|^2 - |\mathbf{v}|^2
\end{aligned}$$

Thus $|\mathbf{u}|^2 - |\mathbf{v}|^2 = 0 \quad\Rightarrow\quad |\mathbf{u}|^2 = |\mathbf{v}|^2 \quad\Rightarrow\quad |\mathbf{u}| = |\mathbf{v}|$ [since $|\mathbf{u}|, |\mathbf{v}| \geq 0$].

30. (a) $\overrightarrow{PQ} = \langle 4,2,3\rangle$ and $\overrightarrow{PR} = \langle 3,3,4\rangle$, so a vector orthogonal to the plane through P, Q, and R is

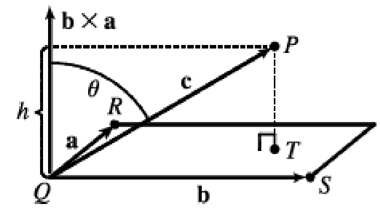
$\overrightarrow{PQ}\times\overrightarrow{PR} = \langle (2)(4)-(3)(3), (3)(3)-(4)(4), (4)(3)-(2)(3)\rangle = \langle -1,-7,6\rangle$ (or any nonzero scalar mutiple thereof).

(b) The area of the parallelogram determined by $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ is $\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = |\langle -1,-7,6\rangle| = \sqrt{1+49+36} = \sqrt{86}$,

so the area of triangle PQR is $\frac{1}{2}\sqrt{86}$.

37. $\mathbf{u}\cdot(\mathbf{v}\times\mathbf{w}) = \begin{vmatrix} 1 & 5 & -2 \\ 3 & -1 & 0 \\ 5 & 9 & -4 \end{vmatrix} = 1\begin{vmatrix} -1 & 0 \\ 9 & -4 \end{vmatrix} - 5\begin{vmatrix} 3 & 0 \\ 5 & -4 \end{vmatrix} + (-2)\begin{vmatrix} 3 & -1 \\ 5 & 9 \end{vmatrix} = 4 + 60 - 64 = 0$, which says that the volume of the parallelepiped determined by $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w}$ is 0, and thus these three vectors are coplanar.

46. (a) The distance between a point and a plane is the length of the perpendicular from the point to the plane, here $\left|\overrightarrow{TP}\right| = d$. But $\overrightarrow{TP}$ is parallel to $\mathbf{b}\times\mathbf{a}$ (because $\mathbf{b}\times\mathbf{a}$ is perpendicular to $\mathbf{b}$ and $\mathbf{a}$) and $d = \left|\overrightarrow{TP}\right| =$ the absolute value of the scalar projection of $\mathbf{c}$ along $\mathbf{b}\times\mathbf{a}$, which is $|\mathbf{c}|\,|\cos\theta|$. (Notice that this is the same setup as the development of the volume of a parallelepiped with $h = |\mathbf{c}|\,|\cos\theta|$). Thus $d = |\mathbf{c}|\,|\cos\theta| = h = V/A$ where $A = |\mathbf{a}\times\mathbf{b}|$, the area of the base. So finally $d = \dfrac{V}{A} = \dfrac{|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|}{|\mathbf{a}\times\mathbf{b}|}$.



(b) $\mathbf{a} = \overrightarrow{QR} = \langle -1, 2, 0\rangle$, $\mathbf{b} = \overrightarrow{QS} = \langle -1, 0, 3\rangle$ and $\mathbf{c} = \overrightarrow{QP} = \langle 1, 1, 4\rangle$. Then

$$\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} -1 & 2 & 0 \\ -1 & 0 & 3 \\ 1 & 1 & 4 \end{vmatrix} = (-1)\begin{vmatrix} 0 & 3 \\ 1 & 4 \end{vmatrix} - 2\begin{vmatrix} -1 & 3 \\ 1 & 4 \end{vmatrix} + 0 = 17$$

and
$$\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1 & 2 & 0 \\ -1 & 0 & 3 \end{vmatrix} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} -1 & 0 \\ -1 & 3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} -1 & 2 \\ -1 & 0 \end{vmatrix}\mathbf{k} = 6\,\mathbf{i} + 3\,\mathbf{j} + 2\,\mathbf{k}$$

Thus $d = \dfrac{|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|}{|\mathbf{a}\times\mathbf{b}|} = \dfrac{17}{\sqrt{36+9+4}} = \dfrac{17}{7}$.

53. (a) No. If $\mathbf{a}\cdot\mathbf{b} = \mathbf{a}\cdot\mathbf{c}$, then $\mathbf{a}\cdot(\mathbf{b}-\mathbf{c}) = 0$, so $\mathbf{a}$ is perpendicular to $\mathbf{b}-\mathbf{c}$, which can happen if $\mathbf{b}\neq\mathbf{c}$. For example, let $\mathbf{a} = \langle 1, 1, 1\rangle$, $\mathbf{b} = \langle 1, 0, 0\rangle$ and $\mathbf{c} = \langle 0, 1, 0\rangle$.

(b) No. If $\mathbf{a}\times\mathbf{b} = \mathbf{a}\times\mathbf{c}$ then $\mathbf{a}\times(\mathbf{b}-\mathbf{c}) = \mathbf{0}$, which implies that $\mathbf{a}$ is parallel to $\mathbf{b}-\mathbf{c}$, which of course can happen if $\mathbf{b}\neq\mathbf{c}$.

(c) Yes. Since $\mathbf{a}\cdot\mathbf{c} = \mathbf{a}\cdot\mathbf{b}$, $\mathbf{a}$ is perpendicular to $\mathbf{b}-\mathbf{c}$, by part (a). From part (b), $\mathbf{a}$ is also parallel to $\mathbf{b}-\mathbf{c}$. Thus since $\mathbf{a}\neq\mathbf{0}$ but is both parallel and perpendicular to $\mathbf{b}-\mathbf{c}$, we have $\mathbf{b}-\mathbf{c} = \mathbf{0}$, so $\mathbf{b} = \mathbf{c}$.

3. For this line, we have $\mathbf{r}_0 = 2\,\mathbf{i} + 2.4\,\mathbf{j} + 3.5\,\mathbf{k}$ and $\mathbf{v} = 3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}$, so a vector equation is $\mathbf{r} = \mathbf{r}_0 + t\,\mathbf{v} = (2\,\mathbf{i} + 2.4\,\mathbf{j} + 3.5\,\mathbf{k}) + t(3\,\mathbf{i} + 2\,\mathbf{j} - \mathbf{k}) = (2+3t)\,\mathbf{i} + (2.4+2t)\,\mathbf{j} + (3.5-t)\,\mathbf{k}$ and parametric equations are $x = 2+3t$, $y = 2.4+2t$, $z = 3.5-t$.

16.

(a) A vector normal to the plane $x - y + 3z = 7$ is $\mathbf{n} = \langle 1, -1, 3 \rangle$, and since the line is to be perpendicular to the plane, $\mathbf{n}$ is also a direction vector for the line. Thus parametric equations of the line are $x = 2 + t$, $y = 4 - t$, $z = 6 + 3t$.

(b) On the xy-plane, $z = 0$. So $z = 6 + 3t = 0 \quad \Rightarrow \quad t = -2$ in the parametric equations of the line, and therefore $x = 0$ and $y = 6$, giving the point of intersection $(0, 6, 0)$. For the yz-plane, $x = 0$ so we get the same point of interesection: $(0, 6, 0)$. For the xz-plane, $y = 0$ which implies $t = 4$, so $x = 6$ and $z = 18$ and the point of intersection is $(6, 0, 18)$.

21.

Since the direction vectors $\langle 1, -2, -3 \rangle$ and $\langle 1, 3, -7 \rangle$ aren't scalar multiples of each other, the lines aren't parallel. Parametric equations of the lines are L_1: $x = 2 + t$, $y = 3 - 2t$, $z = 1 - 3t$ and L_2: $x = 3 + s$, $y = -4 + 3s$, $z = 2 - 7s$. Thus, for the lines to intersect, the three equations $2 + t = 3 + s$, $3 - 2t = -4 + 3s$, and $1 - 3t = 2 - 7s$ must be satisfied simultaneously. Solving the first two equations gives $t = 2$, $s = 1$ and checking, we see that these values do satisfy the third equation, so the lines intersect when $t = 2$ and $s = 1$, that is, at the point $(4, -1, -5)$.

32. Here the vectors $\mathbf{a} = \langle 2, -4, 6 \rangle$ and $\mathbf{b} = \langle 5, 1, 3 \rangle$ lie in the plane, so

$\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -12 - 6, 30 - 6, 2 + 20 \rangle = \langle -18, 24, 22 \rangle$ is a normal vector to the plane and an equation of the plane is $-18(x - 0) + 24(y - 0) + 22(z - 0) = 0$ or $-18x + 24y + 22z = 0$.

35.

If we first find two nonparallel vectors in the plane, their cross product will be a normal vector to the plane. Since the given line lies in the plane, its direction vector $\mathbf{a} = \langle -2, 5, 4 \rangle$ is one vector in the plane. We can verify that the given point $(6, 0, -2)$ does not lie on this line, so to find another nonparallel vector $\mathbf{b}$ which lies in the plane, we can pick any point on the line and find a vector connecting the points. If we put $t = 0$, we see that $(4, 3, 7)$ is on the line, so

$\mathbf{b} = \langle 6 - 4, 0 - 3, -2 - 7 \rangle = \langle 2, -3, -9 \rangle$ and $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle -45 + 12, 8 - 18, 6 - 10 \rangle = \langle -33, -10, -4 \rangle$. Thus, an equation of the plane is $-33(x - 6) - 10(y - 0) - 4[z - (-2)] = 0$ or $33x + 10y + 4z = 190$.

38. The points $(0, -2, 5)$ and $(-1, 3, 1)$ lie in the desired plane, so the vector $\mathbf{v}_1 = \langle -1, 5, -4 \rangle$ connecting them is parallel to the plane. The desired plane is perpendicular to the plane $2z = 5x + 4y$ or $5x + 4y - 2z = 0$ and for perpendicular planes, a normal vector for one plane is parallel to the other plane, so $\mathbf{v}_2 = \langle 5, 4, -2 \rangle$ is also parallel to the desired plane.

A normal vector to the desired plane is $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle -10 + 16, -20 - 2, -4 - 25 \rangle = \langle 6, -22, -29 \rangle$.

Taking $(x_0, y_0, z_0) = (0, -2, 5)$, the equation we are looking for is $6(x - 0) - 22(y + 2) - 29(z - 5) = 0$ or $6x - 22y - 29z = -101$.

40.

$\mathbf{n}_1 = \langle 1, 0, -1 \rangle$ and $\mathbf{n}_2 = \langle 0, 1, 2 \rangle$. Setting $z = 0$, it is easy to see that $(1, 3, 0)$ is a point on the line of intersection of $x - z = 1$ and $y + 2z = 3$. The direction of this line is $\mathbf{v}_1 = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, -2, 1 \rangle$. A second vector parallel to the desired plane is $\mathbf{v}_2 = \langle 1, 1, -2 \rangle$, since it is perpendicular to $x + y - 2z = 1$. Therefore, a normal of the plane in question is $\mathbf{n} = \mathbf{v}_1 \times \mathbf{v}_2 = \langle 4 - 1, 1 + 2, 1 + 2 \rangle = \langle 3, 3, 3 \rangle$, or we can use $\langle 1, 1, 1 \rangle$. Taking $(x_0, y_0, z_0) = (1, 3, 0)$, the equation we are looking for is $(x - 1) + (y - 3) + z = 0 \quad \Leftrightarrow \quad x + y + z = 4$.

57.

(a) To find a point on the line of intersection, set one of the variables equal to a constant, say $z = 0$. (This will fail if the line of intersection does not cross the xy-plane; in that case, try setting x or y equal to 0.) The equations of the two planes reduce to $x + y = 1$ and $x + 2y = 1$. Solving these two equations gives $x = 1$, $y = 0$. Thus a point on the line is $(1, 0, 0)$.
A vector $\mathbf{v}$ in the direction of this intersecting line is perpendicular to the normal vectors of both planes, so we can take $\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, 1 \rangle \times \langle 1, 2, 2 \rangle = \langle 2 - 2, 1 - 2, 2 - 1 \rangle = \langle 0, -1, 1 \rangle$. By Equations 2, parametric equations for the line are $x = 1$, $y = -t$, $z = t$.

(b) The angle between the planes satisfies $\cos\theta = \dfrac{\mathbf{n}_1 \cdot \mathbf{n}_2}{|\mathbf{n}_1|\,|\mathbf{n}_2|} = \dfrac{1 + 2 + 2}{\sqrt{3}\,\sqrt{9}} = \dfrac{5}{3\sqrt{3}}$. Therefore $\theta = \cos^{-1}\left(\dfrac{5}{3\sqrt{3}}\right) \approx 15.8^\circ$.

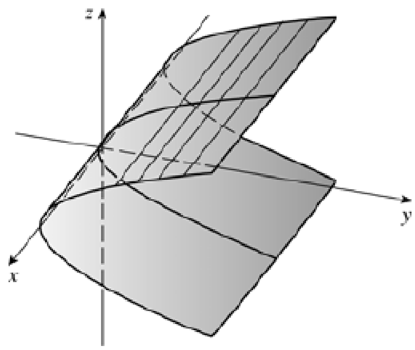
67.

Let P_i have normal vector $\mathbf{n}_i$. Then $\mathbf{n}_1 = \langle 3, 6, -3 \rangle$, $\mathbf{n}_2 = \langle 4, -12, 8 \rangle$, $\mathbf{n}_3 = \langle 3, -9, 6 \rangle$, $\mathbf{n}_4 = \langle 1, 2, -1 \rangle$. Now $\mathbf{n}_1 = 3\mathbf{n}_4$, so $\mathbf{n}_1$ and $\mathbf{n}_4$ are parallel, and hence P_1 and P_4 are parallel; similarly P_2 and P_3 are parallel because $\mathbf{n}_2 = \frac{4}{3}\mathbf{n}_3$. However, $\mathbf{n}_1$ and $\mathbf{n}_2$ are not parallel (so not all four planes are parallel). Notice that the point $(2, 0, 0)$ lies on both P_1 and P_4, so these two planes are identical. The point $\left(\frac{5}{4}, 0, 0\right)$ lies on P_2 but not on P_3, so these are different planes.

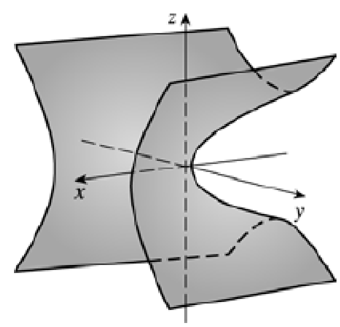
68.

Let L_i have direction vector $\mathbf{v}_i$. Rewrite the symmetric equations for L_3 as $\dfrac{x - 1}{1/2} = \dfrac{y - 1}{-1/4} = \dfrac{z + 1}{1}$; then $\mathbf{v}_1 = \langle 6, -3, 12 \rangle$, $\mathbf{v}_2 = \langle 2, 1, 4 \rangle$, $\mathbf{v}_3 = \left\langle \frac{1}{2}, -\frac{1}{4}, 1 \right\rangle$, and $\mathbf{v}_4 = \langle 4, 2, 8 \rangle$. $\mathbf{v}_1 = 12\mathbf{v}_3$, so L_1 and L_3 are parallel. $\mathbf{v}_4 = 2\mathbf{v}_2$, so L_2 and L_4 are parallel. (Note that L_1 and L_2 are not parallel.) L_1 contains the point $(1, 1, 5)$, but this point does not lie on L_3, so they're not identical. $(3, 1, 5)$ lies on L_4 and also on L_2 (for $t = 1$), so L_2 and L_4 are the same line.

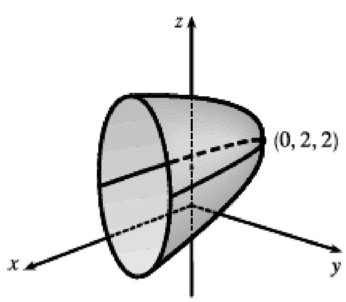
6. Since x is missing, each vertical trace $y = z^2$, $x = k$, is a copy of the same parabola in the plane $x = k$. Thus the surface $y = z^2$ is a parabolic cylinder with rulings parallel to the x-axis.



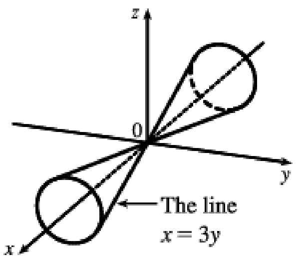
20. $x = y^2 - z^2$. The traces in $x = k$ are $y^2 - z^2 = k$, two intersecting lines when $k = 0$ and a family of hyperbolas for $k \neq 0$ (oriented differently for $k > 0$ than for $k < 0$). The traces in $y = k$ are the parabolas $x = -z^2 + k^2$, opening in the negative x-direction, and the traces in $z = k$ are the parabolas $x = y^2 - k^2$ which open in the positive x-direction. The graph is a hyperbolic paraboloid with saddle point $(0, 0, 0)$.



34. Completing squares in y and z gives
$4(y-2)^2 + (z-2)^2 - x = 0$ or
$\frac{x}{4} = (y-2)^2 + \frac{(z-2)^2}{4}$, an elliptic paraboloid with vertex $(0, 2, 2)$ and axis the horizontal line $y = 2$, $z = 2$.



44. The surface is a right circular cone with vertex at $(0, 0, 0)$ and axis the x-axis. For $x = k \neq 0$, the trace is a circle with center $(k, 0, 0)$ and radius $r = y = \dfrac{x}{3} = \dfrac{k}{3}$. Thus the equation is $(x/3)^2 = y^2 + z^2$ or $x^2 = 9y^2 + 9z^2$.



45. Let $P = (x, y, z)$ be an arbitrary point equidistant from $(-1, 0, 0)$ and the plane $x = 1$. Then the distance from P to $(-1, 0, 0)$ is $\sqrt{(x+1)^2 + y^2 + z^2}$ and the distance from P to the plane $x = 1$ is $|x-1| / \sqrt{1^2} = |x-1|$ (by Equation 12.5.9). So $|x-1| = \sqrt{(x+1)^2 + y^2 + z^2} \quad \Leftrightarrow \quad (x-1)^2 = (x+1)^2 + y^2 + z^2 \quad \Leftrightarrow$ $x^2 - 2x + 1 = x^2 + 2x + 1 + y^2 + z^2 \quad \Leftrightarrow \quad -4x = y^2 + z^2$. Thus the collection of all such points P is a circular paraboloid with vertex at the origin, axis the x-axis, which opens in the negative direction.

46. Let $P = (x, y, z)$ be an arbitrary point whose distance from the x-axis is twice its distance from the yz-plane. The distance from P to the x-axis is $\sqrt{(x-x)^2 + y^2 + z^2} = \sqrt{y^2 + z^2}$ and the distance from P to the yz-plane ($x = 0$) is $|x| / 1 = |x|$. Thus $\sqrt{y^2 + z^2} = 2|x| \quad \Leftrightarrow \quad y^2 + z^2 = 4x^2 \quad \Leftrightarrow \quad x^2 = (y^2/2^2) + (z^2/2^2)$. So the surface is a right circular cone with vertex the origin and axis the x-axis.

49. If (a, b, c) satisfies $z = y^2 - x^2$, then $c = b^2 - a^2$. L_1: $x = a + t$, $y = b + t$, $z = c + 2(b-a)t$, L_2: $x = a + t$, $y = b - t$, $z = c - 2(b+a)t$. Substitute the parametric equations of L_1 into the equation of the hyperbolic paraboloid in order to find the points of intersection: $z = y^2 - x^2 \quad \Rightarrow$ $c + 2(b-a)t = (b+t)^2 - (a+t)^2 = b^2 - a^2 + 2(b-a)t \quad \Rightarrow \quad c = b^2 - a^2$. As this is true for all values of t, L_1 lies on $z = y^2 - x^2$. Performing similar operations with L_2 gives: $z = y^2 - x^2 \quad \Rightarrow$ $c - 2(b+a)t = (b-t)^2 - (a+t)^2 = b^2 - a^2 - 2(b+a)t \quad \Rightarrow \quad c = b^2 - a^2$. This tells us that all of L_2 also lies on $z = y^2 - x^2$.