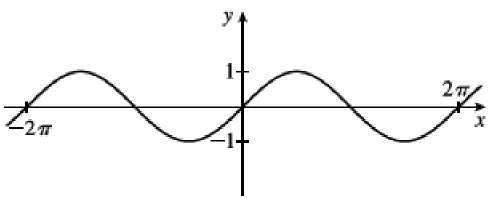
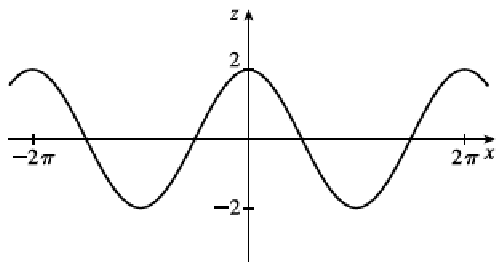
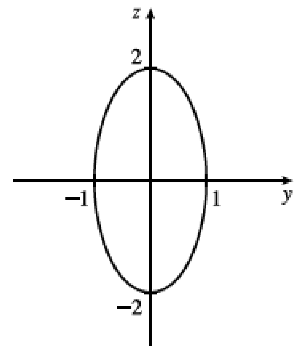
15. The projection of the curve onto the xy-plane is given by $\mathbf{r}(t) = \langle t, \sin t, 0 \rangle$ [we use 0 for the z-component] whose graph is the curve $y = \sin x$, $z = 0$. Similarly, the projection onto the xz-plane is $\mathbf{r}(t) = \langle t, 0, 2\cos t \rangle$, whose graph is the cosine wave $z = 2\cos x$, $y = 0$, and the projection onto the yz-plane is $\mathbf{r}(t) = \langle 0, \sin t, 2\cos t \rangle$ whose graph is the ellipse $y^2 + \frac{1}{4}z^2 = 1$, $x = 0$.



xy-plane

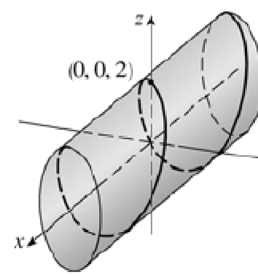


xz-plane



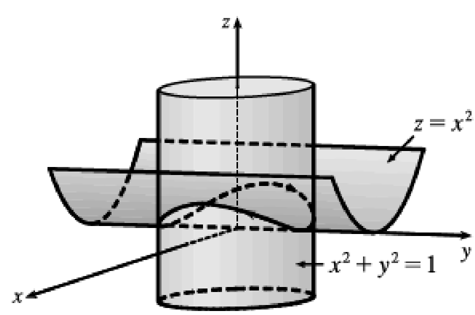
yz-plane

From the projection onto the yz-plane we see that the curve lies on an elliptical cylinder with axis the x-axis. The other two projections show that the curve oscillates both vertically and horizontally as we move in the x-direction, suggesting that the curve is an elliptical helix that spirals along the cylinder.



20. Taking $\mathbf{r}_0 = \langle a, b, c \rangle$ and $\mathbf{r}_1 = \langle u, v, w \rangle$, we have
$\mathbf{r}(t) = (1-t)\,\mathbf{r}_0 + t\,\mathbf{r}_1 = (1-t)\,\langle a, b, c \rangle + t\,\langle u, v, w \rangle$, $0 \le t \le 1$ or $\mathbf{r}(t) = \langle a + (u-a)t, b + (v-b)t, c + (w-c)t \rangle$, $0 \le t \le 1$. Parametric equations are $x = a + (u-a)t$, $y = b + (v-b)t$, $z = c + (w-c)t$, $0 \le t \le 1$.

28. Here $x^2 = \sin^2 t = z$ and $x^2 + y^2 = \sin^2 t + \cos^2 t = 1$, so the curve is contained in the intersection of the parabolic cylinder $z = x^2$ with the circular cylinder $x^2 + y^2 = 1$. We get the complete intersection for $0 \le t \le 2\pi$.



30. Parametric equations for the helix are $x = \sin t$, $y = \cos t$, $z = t$. Substituting into the equation of the sphere gives $\sin^2 t + \cos^2 t + t^2 = 5 \;\Rightarrow\; 1 + t^2 = 5 \;\Rightarrow\; t = \pm 2$. Since $\mathbf{r}(2) = \langle \sin 2, \cos 2, 2 \rangle$ and $\mathbf{r}(-2) = \langle \sin(-2), \cos(-2), -2 \rangle$, the points of intersection are $(\sin 2, \cos 2, 2) \approx (0.909, -0.416, 2)$ and $(\sin(-2), \cos(-2), -2) \approx (-0.909, -0.416, -2)$.

41.
Both equations are solved for z, so we can substitute to eliminate z: $\sqrt{x^2+y^2} = 1+y \quad \Rightarrow \quad x^2+y^2 = 1+2y+y^2 \quad \Rightarrow$ $x^2 = 1+2y \quad \Rightarrow \quad y = \frac{1}{2}(x^2-1)$. We can form parametric equations for the curve C of intersection by choosing a parameter $x = t$, then $y = \frac{1}{2}(t^2-1)$ and $z = 1+y = 1+\frac{1}{2}(t^2-1) = \frac{1}{2}(t^2+1)$. Thus a vector function representing C is $\mathbf{r}(t) = t\,\mathbf{i} + \frac{1}{2}(t^2-1)\,\mathbf{j} + \frac{1}{2}(t^2+1)\,\mathbf{k}$.

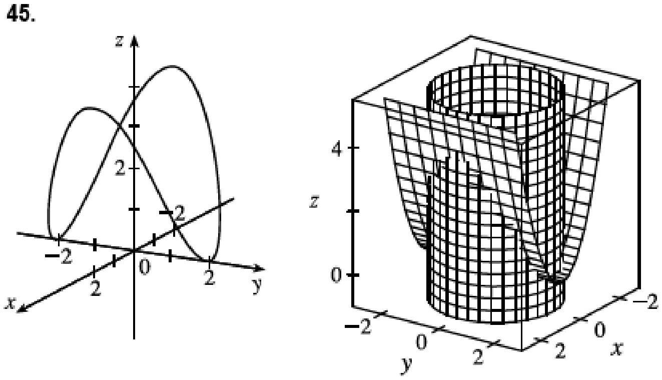
43. The projection of the curve C of intersection onto the xy-plane is the circle $x^2+y^2=1$, $z=0$, so we can write $x=\cos t$, $y=\sin t$, $0 \le t \le 2\pi$. Since C also lies on the surface $z = x^2-y^2$, we have $z = x^2-y^2 = \cos^2 t - \sin^2 t$ or $\cos 2t$. Thus parametric equations for C are $x=\cos t$, $y=\sin t$, $z=\cos 2t$, $0 \le t \le 2\pi$, and the corresponding vector function is $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sin t\,\mathbf{j} + \cos 2t\,\mathbf{k}$, $0 \le t \le 2\pi$.

44.
The projection of the curve C of intersection onto the xz-plane is the circle $x^2+z^2=1$, $y=0$, so we can write $x=\cos t$, $z=\sin t$, $0 \le t \le 2\pi$. C also lies on the surface $x^2+y^2+4z^2=4$, and since $y \ge 0$ we can write

$$y = \sqrt{4-x^2-4z^2} = \sqrt{4-\cos^2 t - 4\sin^2 t} = \sqrt{4-\cos^2 t - 4(1-\cos^2 t)} = \sqrt{3\cos^2 t} = \sqrt{3}\,|\cos t\,|$$

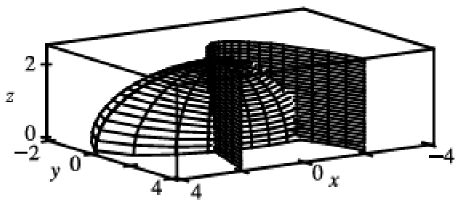
Thus parametric equations for C are $x=\cos t$, $y=\sqrt{3}\,|\cos t\,|$, $z=\sin t$, $0 \le t \le 2\pi$, and the corresponding vector function is $\mathbf{r}(t) = \cos t\,\mathbf{i} + \sqrt{3}\,|\cos t\,|\,\mathbf{j} + \sin t\,\mathbf{k}$, $0 \le t \le 2\pi$.

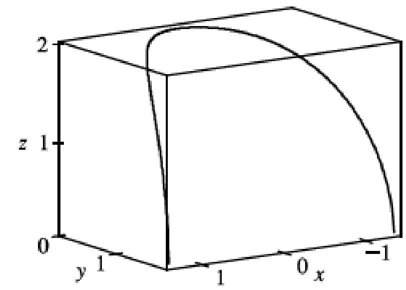
45.



The projection of the curve C of intersection onto the xy-plane is the circle $x^2+y^2=4$, $z=0$. Then we can write $x = 2\cos t,\ y = 2\sin t,\ 0 \le t \le 2\pi$. Since C also lies on the surface $z = x^2$, we have $z = x^2 = (2\cos t)^2 = 4\cos^2 t$. Then parametric equations for C are $x = 2\cos t,\ y = 2\sin t,\ z = 4\cos^2 t,\ 0 \le t \le 2\pi$.

46.





$$x = t \quad \Rightarrow \quad y = t^2 \quad \Rightarrow \quad 4z^2 = 16 - x^2 - 4y^2 = 16 - t^2 - 4t^4 \quad \Rightarrow \quad z = \sqrt{4 - \left(\tfrac{1}{2}t\right)^2 - t^4}.$$

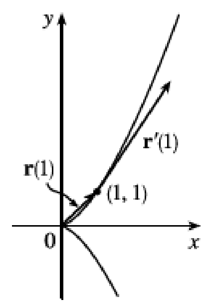
Note that z is positive because the intersection is with the top half of the ellipsoid. Hence the curve is given by $x = t,\ y = t^2,\ z = \sqrt{4 - \frac{1}{4}t^2 - t^4}$.

47.

For the particles to collide, we require $\mathbf{r}_1(t) = \mathbf{r}_2(t) \quad \Leftrightarrow \quad \langle t^2, 7t - 12, t^2 \rangle = \langle 4t - 3, t^2, 5t - 6 \rangle$. Equating components gives $t^2 = 4t - 3$, $7t - 12 = t^2$, and $t^2 = 5t - 6$. From the first equation, $t^2 - 4t + 3 = 0 \quad \Leftrightarrow \quad (t - 3)(t - 1) = 0$ so $t = 1$ or $t = 3$. $t = 1$ does not satisfy the other two equations, but $t = 3$ does. The particles collide when $t = 3$, at the point $(9, 9, 9)$.

4. Since $x = t^2 = (t^3)^{2/3} = y^{2/3}$, the curve is the graph of $x = y^{2/3}$.

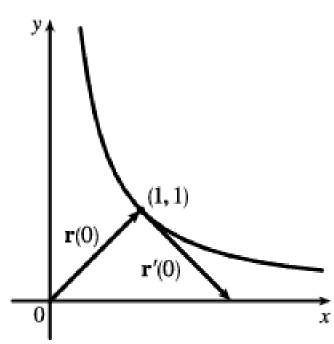
(a), (c)



(b) $\mathbf{r}'(t) = \langle 2t, 3t^2 \rangle$, $\mathbf{r}'(1) = \langle 2, 3 \rangle$

6. Since $y = e^{-t} = \dfrac{1}{e^t} = \dfrac{1}{x}$ the curve is part of the hyperbola $y = \dfrac{1}{x}$. Note that $x > 0$, $y > 0$.

(a), (c)



(b) $\mathbf{r}'(t) = e^t\,\mathbf{i} - e^{-t}\,\mathbf{j}$, $\mathbf{r}'(0) = \mathbf{i} - \mathbf{j}$

14.
$$\begin{aligned}\mathbf{r}'(t) &= [at(-3\sin 3t) + a\cos 3t]\,\mathbf{i} + b \cdot 3\sin^2 t\cos t\,\mathbf{j} + c \cdot 3\cos^2 t(-\sin t)\,\mathbf{k} \\ &= (a\cos 3t - 3at\sin 3t)\,\mathbf{i} + 3b\sin^2 t\ \cos t\,\mathbf{j} - 3c\cos^2 t\ \sin t\,\mathbf{k}\end{aligned}$$

18. $\mathbf{r}'(t) = \langle 3t^2+3, 2t, 3\rangle \quad\Rightarrow\quad \mathbf{r}'(1) = \langle 6,2,3\rangle$. Thus

$$\mathbf{T}(1) = \frac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \frac{1}{\sqrt{6^2+2^2+3^2}}\langle 6,2,3\rangle = \tfrac{1}{7}\langle 6,2,3\rangle = \left\langle \tfrac{6}{7}, \tfrac{2}{7}, \tfrac{3}{7}\right\rangle.$$

34.
To find the point of intersection, we must find the values of t and s which satisfy the following three equations simultaneously: $t = 3-s$, $1-t = s-2$, $3+t^2 = s^2$. Solving the last two equations gives $t=1$, $s=2$ (check these in the first equation). Thus the point of intersection is $(1,0,4)$. To find the angle θ of intersection, we proceed as in Exercise 33. The tangent vectors to the respective curves at $(1,0,4)$ are $\mathbf{r}_1'(1) = \langle 1,-1,2\rangle$ and $\mathbf{r}_2'(2) = \langle -1,1,4\rangle$. So $\cos\theta = \frac{1}{\sqrt{6}\sqrt{18}}(-1-1+8) = \frac{6}{6\sqrt{3}} = \frac{1}{\sqrt{3}}$ and $\theta = \cos^{-1}\left(\frac{1}{\sqrt{3}}\right) \approx 55^\circ$.

Note: In Exercise 33, the curves intersect when the value of both parameters is zero. However, as seen in this exercise, it is not necessary for the parameters to be of equal value at the point of intersection.

41. $\mathbf{r}'(t) = 2t\,\mathbf{i} + 3t^2\,\mathbf{j} + \sqrt{t}\,\mathbf{k} \quad\Rightarrow\quad \mathbf{r}(t) = t^2\,\mathbf{i} + t^3\,\mathbf{j} + \frac{2}{3}t^{3/2}\,\mathbf{k} + \mathbf{C}$, where $\mathbf{C}$ is a constant vector.

But $\mathbf{i}+\mathbf{j} = \mathbf{r}(1) = \mathbf{i}+\mathbf{j}+\frac{2}{3}\mathbf{k}+\mathbf{C}$. Thus $\mathbf{C} = -\frac{2}{3}\mathbf{k}$ and $\mathbf{r}(t) = t^2\,\mathbf{i} + t^3\,\mathbf{j} + \left(\frac{2}{3}t^{3/2} - \frac{2}{3}\right)\mathbf{k}$.

3. $\mathbf{r}(t) = \sqrt{2}\,t\,\mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k} \quad\Rightarrow\quad \mathbf{r}'(t) = \sqrt{2}\,\mathbf{i} + e^t\mathbf{j} - e^{-t}\mathbf{k} \quad\Rightarrow$

$|\mathbf{r}'(t)| = \sqrt{\left(\sqrt{2}\right)^2 + (e^t)^2 + (-e^{-t})^2} = \sqrt{2+e^{2t}+e^{-2t}} = \sqrt{(e^t+e^{-t})^2} = e^t + e^{-t}$ [since $e^t + e^{-t} > 0$].

Then $L = \int_0^1 |\mathbf{r}'(t)|\,dt = \int_0^1 (e^t + e^{-t})\,dt = \left[e^t - e^{-t}\right]_0^1 = e - e^{-1}$.

6. $\mathbf{r}(t) = 12t\,\mathbf{i} + 8t^{3/2}\,\mathbf{j} + 3t^2\,\mathbf{k} \quad\Rightarrow\quad \mathbf{r}'(t) = 12\,\mathbf{i} + 12\sqrt{t}\,\mathbf{j} + 6t\,\mathbf{k} \quad\Rightarrow$

$|\mathbf{r}'(t)| = \sqrt{144+144t+36t^2} = \sqrt{36(t+2)^2} = 6\,|t+2| = 6(t+2)$ for $0 \le t \le 1$. Then

$L = \int_0^1 |\mathbf{r}'(t)|\,dt = \int_0^1 6(t+2)\,dt = \Big[3t^2+12t\Big]_0^1 = 15$.

14. $\mathbf{r}(t) = e^{2t}\cos 2t\,\mathbf{i} + 2\,\mathbf{j} + e^{2t}\sin 2t\,\mathbf{k} \quad\Rightarrow\quad \mathbf{r}'(t) = 2e^{2t}(\cos 2t - \sin 2t)\,\mathbf{i} + 2e^{2t}(\cos 2t + \sin 2t)\,\mathbf{k}$,

$\frac{ds}{dt} = |\mathbf{r}'(t)| = 2e^{2t}\sqrt{(\cos 2t - \sin 2t)^2 + (\cos 2t + \sin 2t)^2} = 2e^{2t}\sqrt{2\cos^2 2t + 2\sin^2 2t} = 2\sqrt{2}\,e^{2t}$.

$s = s(t) = \int_0^t |\mathbf{r}'(u)|\,du = \int_0^t 2\sqrt{2}\,e^{2u}\,du = \sqrt{2}\,e^{2u}\Big]_0^t = \sqrt{2}\,(e^{2t}-1) \quad\Rightarrow\quad \frac{s}{\sqrt{2}} + 1 = e^{2t} \quad\Rightarrow\quad t = \frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)$.

Substituting, we have

$$\begin{aligned}\mathbf{r}(t(s)) &= e^{2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)}\cos 2\left(\tfrac{1}{2}\ln\left(\tfrac{s}{\sqrt{2}}+1\right)\right)\mathbf{i} + 2\,\mathbf{j} + e^{2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)}\sin 2\left(\tfrac{1}{2}\ln\left(\tfrac{s}{\sqrt{2}}+1\right)\right)\mathbf{k} \\ &= \left(\tfrac{s}{\sqrt{2}}+1\right)\cos\left(\ln\left(\tfrac{s}{\sqrt{2}}+1\right)\right)\mathbf{i} + 2\,\mathbf{j} + \left(\tfrac{s}{\sqrt{2}}+1\right)\sin\left(\ln\left(\tfrac{s}{\sqrt{2}}+1\right)\right)\mathbf{k}\end{aligned}$$

19.

(a) $\mathbf{r}(t) = \left\langle \sqrt{2}\, t, e^t, e^{-t} \right\rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle \quad \Rightarrow \quad |\mathbf{r}'(t)| = \sqrt{2 + e^{2t} + e^{-2t}} = \sqrt{(e^t + e^{-t})^2} = e^t + e^{-t}$.

Then

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{e^t + e^{-t}} \left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle = \frac{1}{e^{2t}+1} \left\langle \sqrt{2}\, e^t, e^{2t}, -1 \right\rangle \quad \left[\text{after multiplying by } \frac{e^t}{e^t}\right] \quad \text{and}$$

$$\mathbf{T}'(t) = \frac{1}{e^{2t}+1} \left\langle \sqrt{2}\, e^t, 2e^{2t}, 0 \right\rangle - \frac{2e^{2t}}{\left(e^{2t}+1\right)^2} \left\langle \sqrt{2}\, e^t, e^{2t}, -1 \right\rangle$$

$$= \frac{1}{(e^{2t}+1)^2} \left[(e^{2t}+1) \left\langle \sqrt{2}\, e^t, 2e^{2t}, 0 \right\rangle - 2e^{2t} \left\langle \sqrt{2}\, e^t, e^{2t}, -1 \right\rangle \right] = \frac{1}{(e^{2t}+1)^2} \left\langle \sqrt{2}\, e^t \left(1 - e^{2t}\right), 2e^{2t}, 2e^{2t} \right\rangle$$

Then

$$|\mathbf{T}'(t)| = \frac{1}{(e^{2t}+1)^2} \sqrt{2e^{2t}(1 - 2e^{2t} + e^{4t}) + 4e^{4t} + 4e^{4t}} = \frac{1}{\left(e^{2t}+1\right)^2} \sqrt{2e^{2t}(1 + 2e^{2t} + e^{4t})}$$

$$= \frac{1}{(e^{2t}+1)^2} \sqrt{2e^{2t} \left(1 + e^{2t}\right)^2} = \frac{\sqrt{2}\, e^t (1 + e^{2t})}{(e^{2t}+1)^2} = \frac{\sqrt{2}\, e^t}{e^{2t}+1}$$

Therefore

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{e^{2t}+1}{\sqrt{2}\, e^t} \frac{1}{(e^{2t}+1)^2} \left\langle \sqrt{2}\, e^t (1 - e^{2t}), 2e^{2t}, 2e^{2t} \right\rangle$$

$$= \frac{1}{\sqrt{2}\, e^t (e^{2t}+1)} \left\langle \sqrt{2}\, e^t (1 - e^{2t}), 2e^{2t}, 2e^{2t} \right\rangle = \frac{1}{e^{2t}+1} \left\langle 1 - e^{2t}, \sqrt{2}\, e^t, \sqrt{2}\, e^t \right\rangle$$

(b) $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{\sqrt{2}\, e^t}{e^{2t}+1} \cdot \dfrac{1}{e^t + e^{-t}} = \dfrac{\sqrt{2}\, e^t}{e^{3t} + 2e^t + e^{-t}} = \dfrac{\sqrt{2}\, e^{2t}}{e^{4t} + 2e^{2t} + 1} = \dfrac{\sqrt{2}\, e^{2t}}{(e^{2t}+1)^2}$

20. (a) $\mathbf{r}(t) = \left\langle t, \frac{1}{2}t^2, t^2 \right\rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle 1, t, 2t \rangle \quad \Rightarrow \quad |\mathbf{r}'(t)| = \sqrt{1 + t^2 + 4t^2} = \sqrt{1 + 5t^2}$. Then

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \frac{1}{\sqrt{1+5t^2}} \ \langle 1, t, 2t \rangle.$$

$$\mathbf{T}'(t) = \frac{-5t}{(1+5t^2)^{3/2}} \ \langle 1, t, 2t \rangle + \frac{1}{\sqrt{1+5t^2}} \ \langle 0, 1, 2 \rangle \qquad \text{[by Formula 3 of Theorem 13.2.3]}$$

$$= \frac{1}{(1+5t^2)^{3/2}} \ \left(\left\langle -5t, -5t^2, -10t^2 \right\rangle + \left\langle 0, 1 + 5t^2, 2 + 10t^2 \right\rangle \right) = \frac{1}{(1+5t^2)^{3/2}} \ \langle -5t, 1, 2 \rangle$$

$$|\mathbf{T}'(t)| = \frac{1}{(1+5t^2)^{3/2}} \sqrt{25t^2 + 1 + 4} = \frac{1}{(1+5t^2)^{3/2}} \sqrt{25t^2 + 5} = \frac{\sqrt{5}\,\sqrt{5t^2+1}}{(1+5t^2)^{3/2}} = \frac{\sqrt{5}}{1+5t^2}$$

Thus $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \dfrac{1+5t^2}{\sqrt{5}} \cdot \dfrac{1}{(1+5t^2)^{3/2}} \ \langle -5t, 1, 2 \rangle = \dfrac{1}{\sqrt{5+25t^2}} \ \langle -5t, 1, 2 \rangle$.

(b) $\kappa(t) = \dfrac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \dfrac{\sqrt{5}/(1+5t^2)}{\sqrt{1+5t^2}} = \dfrac{\sqrt{5}}{(1+5t^2)^{3/2}}$

22. $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + e^t\,\mathbf{k} \quad \Rightarrow \quad \mathbf{r}'(t) = \mathbf{i} + 2t\,\mathbf{j} + e^t\,\mathbf{k}, \quad \mathbf{r}''(t) = 2\,\mathbf{j} + e^t\,\mathbf{k}$,

$|\mathbf{r}'(t)| = \sqrt{1^2 + (2t)^2 + (e^t)^2} = \sqrt{1 + 4t^2 + e^{2t}}, \quad \mathbf{r}'(t) \times \mathbf{r}''(t) = (2t-2)e^t\,\mathbf{i} - e^t\,\mathbf{j} + 2\,\mathbf{k}$,

$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \sqrt{[(2t-2)e^t]^2 + (-e^t)^2 + 2^2} = \sqrt{(2t-2)^2e^{2t} + e^{2t} + 4} = \sqrt{(4t^2 - 8t + 5)e^{2t} + 4}$.

Then $\kappa(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \dfrac{\sqrt{(4t^2 - 8t + 5)e^{2t} + 4}}{\left(\sqrt{1 + 4t^2 + e^{2t}}\right)^3} = \dfrac{\sqrt{(4t^2 - 8t + 5)e^{2t} + 4}}{(1 + 4t^2 + e^{2t})^{3/2}}$.

23. $\mathbf{r}(t) = 3t\,\mathbf{i} + 4\sin t\,\mathbf{j} + 4\cos t\,\mathbf{k} \quad \Rightarrow \quad \mathbf{r}'(t) = 3\,\mathbf{i} + 4\cos t\,\mathbf{j} - 4\sin t\,\mathbf{k}, \quad \mathbf{r}''(t) = -4\sin t\,\mathbf{j} - 4\cos t\,\mathbf{k}$,

$|\mathbf{r}'(t)| = \sqrt{9 + 16\cos^2 t + 16\sin^2 t} = \sqrt{9 + 16} = 5, \quad \mathbf{r}'(t) \times \mathbf{r}''(t) = -16\,\mathbf{i} + 12\cos t\,\mathbf{j} - 12\sin t\,\mathbf{k}$,

$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = \sqrt{256 + 144\cos^2 t + 144\sin^2 t} = \sqrt{400} = 20$. Then $\kappa(t) = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = \dfrac{20}{5^3} = \dfrac{4}{25}$.

25. $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle$. The point $(1, 1, 1)$ corresponds to $t = 1$, and $\mathbf{r}'(1) = \langle 1, 2, 3 \rangle \quad \Rightarrow$

$|\mathbf{r}'(1)| = \sqrt{1 + 4 + 9} = \sqrt{14}$. $\mathbf{r}''(t) = \langle 0, 2, 6t \rangle \quad \Rightarrow \quad \mathbf{r}''(1) = \langle 0, 2, 6 \rangle$. $\mathbf{r}'(1) \times \mathbf{r}''(1) = \langle 6, -6, 2 \rangle$, so

$|\mathbf{r}'(1) \times \mathbf{r}''(1)| = \sqrt{36 + 36 + 4} = \sqrt{76}$. Then $\kappa(1) = \dfrac{|\mathbf{r}'(1) \times \mathbf{r}''(1)|}{|\mathbf{r}'(1)|^3} = \dfrac{\sqrt{76}}{\sqrt{14}^3} = \dfrac{1}{7}\sqrt{\dfrac{19}{14}}$.

31. Since $y' = y'' = e^x$, the curvature is $\kappa(x) = \dfrac{|y''(x)|}{[1 + (y'(x))^2]^{3/2}} = \dfrac{e^x}{(1 + e^{2x})^{3/2}} = e^x(1 + e^{2x})^{-3/2}$.

To find the maximum curvature, we first find the critical numbers of $\kappa(x)$:

$\kappa'(x) = e^x(1 + e^{2x})^{-3/2} + e^x\left(-\frac{3}{2}\right)(1 + e^{2x})^{-5/2}(2e^{2x}) = e^x\dfrac{1 + e^{2x} - 3e^{2x}}{(1 + e^{2x})^{5/2}} = e^x\dfrac{1 - 2e^{2x}}{(1 + e^{2x})^{5/2}}$.

$\kappa'(x) = 0$ when $1 - 2e^{2x} = 0$, so $e^{2x} = \frac{1}{2}$ or $x = -\frac{1}{2}\ln 2$. And since $1 - 2e^{2x} > 0$ for $x < -\frac{1}{2}\ln 2$ and $1 - 2e^{2x} < 0$ for $x > -\frac{1}{2}\ln 2$, the maximum curvature is attained at the point $\left(-\frac{1}{2}\ln 2, e^{(-\ln 2)/2}\right) = \left(-\frac{1}{2}\ln 2, \frac{1}{\sqrt{2}}\right)$.

Since $\lim\limits_{x \to \infty} e^x(1 + e^{2x})^{-3/2} = 0$, $\kappa(x)$ approaches 0 as $x \to \infty$.

38.

Notice that the curve a is highest for the same x-values at which curve b is turning more sharply, and a is 0 or near 0 where b is nearly straight. So, a must be the graph of $y = \kappa(x)$, and b is the graph of $y = f(x)$.

39.

Notice that the curve b has two inflection points at which the graph appears almost straight. We would expect the curvature to be 0 or nearly 0 at these values, but the curve a isn't near 0 there. Thus, a must be the graph of $y = f(x)$ rather than the graph of curvature, and b is the graph of $y = \kappa(x)$.

42. Here $\mathbf{r}(t) = \langle f(t), g(t)\rangle$, $\quad \mathbf{r}'(t) = \langle f'(t), g'(t)\rangle$, $\quad \mathbf{r}''(t) = \langle f''(t), g''(t)\rangle$,

$|\mathbf{r}'(t)|^3 = \left[\sqrt{(f'(t))^2 + (g'(t))^2}\,\right]^3 = [(f'(t))^2 + (g'(t))^2]^{3/2} = (\dot{x}^2 + \dot{y}^2)^{3/2}$, and

$|\mathbf{r}'(t) \times \mathbf{r}''(t)| = |\langle 0, 0, f'(t)\,g''(t) - f''(t)\,g'(t)\rangle| = \left[(\dot{x}\ddot{y} - \ddot{x}\dot{y})^2\right]^{1/2} = |\dot{x}\ddot{y} - \dot{y}\ddot{x}|$. Thus $\kappa(t) = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{[\dot{x}^2 + \dot{y}^2]^{3/2}}$.

47. $\left(1, \frac{2}{3}, 1\right)$ corresponds to $t = 1$. $\quad \mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|} = \dfrac{\langle 2t, 2t^2, 1\rangle}{\sqrt{4t^2 + 4t^4 + 1}} = \dfrac{\langle 2t, 2t^2, 1\rangle}{2t^2 + 1}$, so $\mathbf{T}(1) = \left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right\rangle$.

$$\begin{aligned}\mathbf{T}'(t) &= -4t(2t^2+1)^{-2}\left\langle 2t, 2t^2, 1\right\rangle + (2t^2+1)^{-1}\left\langle 2, 4t, 0\right\rangle \qquad \text{[by Formula 3 of Theorem 13.2.3]}\\ &= (2t^2+1)^{-2}\left\langle -8t^2 + 4t^2 + 2, -8t^3 + 8t^3 + 4t, -4t\right\rangle = 2(2t^2+1)^{-2}\left\langle 1 - 2t^2, 2t, -2t\right\rangle\end{aligned}$$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|} = \frac{2(2t^2+1)^{-2}\left\langle 1-2t^2, 2t, -2t\right\rangle}{2(2t^2+1)^{-2}\sqrt{(1-2t^2)^2 + (2t)^2 + (-2t)^2}} = \frac{\left\langle 1-2t^2, 2t, -2t\right\rangle}{\sqrt{1 - 4t^2 + 4t^4 + 8t^2}} = \frac{\left\langle 1-2t^2, 2t, -2t\right\rangle}{1 + 2t^2}$$

$\mathbf{N}(1) = \left\langle -\frac{1}{3}, \frac{2}{3}, -\frac{2}{3}\right\rangle$ and $\mathbf{B}(1) = \mathbf{T}(1) \times \mathbf{N}(1) = \left\langle -\frac{4}{9} - \frac{2}{9}, -\left(-\frac{4}{9} + \frac{1}{9}\right), \frac{4}{9} + \frac{2}{9}\right\rangle = \left\langle -\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right\rangle$.

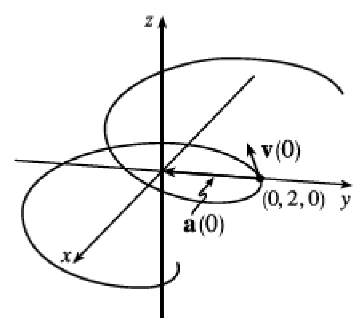
8. $\mathbf{r}(t) = t\,\mathbf{i} + 2\cos t\,\mathbf{j} + \sin t\,\mathbf{k} \quad \Rightarrow \qquad$ At $t = 0$:

$\mathbf{v}(t) = \mathbf{i} - 2\sin t\,\mathbf{j} + \cos t\,\mathbf{k} \qquad \mathbf{v}(0) = \mathbf{i} + \mathbf{k}$

$\mathbf{a}(t) = -2\cos t\,\mathbf{j} - \sin t\,\mathbf{k} \qquad \mathbf{a}(0) = -2\,\mathbf{j}$

$|\mathbf{v}(t)| = \sqrt{1 + 4\sin^2 t + \cos^2 t} = \sqrt{2 + 3\sin^2 t}$

Since $y^2/4 + z^2 = 1$, $x = t$, the path of the particle is an elliptical helix about the x-axis.



13. $\mathbf{r}(t) = e^t\langle \cos t, \sin t, t\rangle \quad \Rightarrow$

$\mathbf{v}(t) = \mathbf{r}'(t) = e^t\langle \cos t, \sin t, t\rangle + e^t\langle -\sin t, \cos t, 1\rangle = e^t\langle \cos t - \sin t, \sin t + \cos t, t + 1\rangle$

$$\begin{aligned}\mathbf{a}(t) = \mathbf{v}'(t) &= e^t\langle \cos t - \sin t - \sin t - \cos t, \sin t + \cos t + \cos t - \sin t, t + 1 + 1\rangle\\ &= e^t\langle -2\sin t, 2\cos t, t + 2\rangle\end{aligned}$$

$$\begin{aligned}|\mathbf{v}(t)| &= e^t\sqrt{\cos^2 t + \sin^2 t - 2\cos t\sin t + \sin^2 t + \cos^2 t + 2\sin t\cos t + t^2 + 2t + 1}\\ &= e^t\sqrt{t^2 + 2t + 3}\end{aligned}$$

15. $\mathbf{a}(t) = \mathbf{i} + 2\,\mathbf{j} \quad \Rightarrow \quad \mathbf{v}(t) = \int \mathbf{a}(t)\,dt = \int(\mathbf{i} + 2\,\mathbf{j})\,dt = t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{C}$ and $\mathbf{k} = \mathbf{v}(0) = \mathbf{C}$,

so $\mathbf{C} = \mathbf{k}$ and $\mathbf{v}(t) = t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k}$. $\quad \mathbf{r}(t) = \int \mathbf{v}(t)\,dt = \int(t\,\mathbf{i} + 2t\,\mathbf{j} + \mathbf{k})\,dt = \frac{1}{2}t^2\,\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k} + \mathbf{D}$.

But $\mathbf{i} = \mathbf{r}(0) = \mathbf{D}$, so $\mathbf{D} = \mathbf{i}$ and $\mathbf{r}(t) = \left(\frac{1}{2}t^2 + 1\right)\mathbf{i} + t^2\,\mathbf{j} + t\,\mathbf{k}$.

20. Since $\mathbf{r}(t) = t^3\,\mathbf{i} + t^2\,\mathbf{j} + t^3\,\mathbf{k}$, $\mathbf{a}(t) = \mathbf{r}''(t) = 6t\,\mathbf{i} + 2\,\mathbf{j} + 6t\,\mathbf{k}$. By Newton's Second Law,

$\mathbf{F}(t) = m\,\mathbf{a}(t) = 6mt\,\mathbf{i} + 2m\,\mathbf{j} + 6mt\,\mathbf{k}$ is the required force.

23.
$|\mathbf{v}(0)| = 200$ m/s and, since the angle of elevation is 60°, a unit vector in the direction of the velocity is

$(\cos 60^\circ)\mathbf{i} + (\sin 60^\circ)\mathbf{j} = \frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}$. Thus $\mathbf{v}(0) = 200\left(\frac{1}{2}\mathbf{i} + \frac{\sqrt{3}}{2}\mathbf{j}\right) = 100\,\mathbf{i} + 100\sqrt{3}\,\mathbf{j}$ and if we set up the axes so that the projectile starts at the origin, then $\mathbf{r}(0) = \mathbf{0}$. Ignoring air resistance, the only force is that due to gravity, so

$\mathbf{F}(t) = m\mathbf{a}(t) = -mg\,\mathbf{j}$ where $g \approx 9.8$ m/s^2. Thus $\mathbf{a}(t) = -9.8\,\mathbf{j}$ and, integrating, we have $\mathbf{v}(t) = -9.8t\,\mathbf{j} + \mathbf{C}$. But

$100\,\mathbf{i} + 100\sqrt{3}\,\mathbf{j} = \mathbf{v}(0) = \mathbf{C}$, so $\mathbf{v}(t) = 100\,\mathbf{i} + \left(100\sqrt{3} - 9.8t\right)\mathbf{j}$ and then (integrating again)

$\mathbf{r}(t) = 100\,t\,\mathbf{i} + \left(100\sqrt{3}\,t - 4.9t^2\right)\mathbf{j} + \mathbf{D}$ where $\mathbf{0} = \mathbf{r}(0) = \mathbf{D}$. Thus the position function of the projectile is

$\mathbf{r}(t) = 100\,t\,\mathbf{i} + \left(100\sqrt{3}\,t - 4.9t^2\right)\mathbf{j}$.

(a) Parametric equations for the projectile are $x(t) = 100t$, $y(t) = 100\sqrt{3}\,t - 4.9t^2$. The projectile reaches the ground when

$y(t) = 0$ (and $t > 0$) $\Rightarrow$ $100\sqrt{3}\,t - 4.9t^2 = t\left(100\sqrt{3} - 4.9t\right) = 0$ $\Rightarrow$ $t = \frac{100\sqrt{3}}{4.9} \approx 35.3$ s. So the range is

$x\left(\frac{100\sqrt{3}}{4.9}\right) = 100\left(\frac{100\sqrt{3}}{4.9}\right) \approx 3535$ m.

(b) The maximum height is reached when $y(t)$ has a critical number (or equivalently, when the vertical component of velocity is 0): $y'(t) = 0$ $\Rightarrow$ $100\sqrt{3} - 9.8t = 0$ $\Rightarrow$ $t = \frac{100\sqrt{3}}{9.8} \approx 17.7$ s. Thus the maximum height is

$y\left(\frac{100\sqrt{3}}{9.8}\right) = 100\sqrt{3}\left(\frac{100\sqrt{3}}{9.8}\right) - 4.9\left(\frac{100\sqrt{3}}{9.8}\right)^2 \approx 1531$ m.

(c) From part (a), impact occurs at $t = \frac{100\sqrt{3}}{4.9}$ s. Thus, the velocity at impact is

$\mathbf{v}\left(\frac{100\sqrt{3}}{4.9}\right) = 100\,\mathbf{i} + \left[100\sqrt{3} - 9.8\left(\frac{100\sqrt{3}}{4.9}\right)\right]\mathbf{j} = 100\,\mathbf{i} - 100\sqrt{3}\,\mathbf{j}$ and the speed is

$\left|\mathbf{v}\left(\frac{100\sqrt{3}}{4.9}\right)\right| = \sqrt{10{,}000 + 30{,}000} = 200$ m/s.

25.
As in Example 5, $\mathbf{r}(t) = (v_0\cos 45^\circ)t\,\mathbf{i} + \left[(v_0\sin 45^\circ)t - \frac{1}{2}gt^2\right]\mathbf{j} = \frac{1}{2}\left[v_0\sqrt{2}\,t\,\mathbf{i} + \left(v_0\sqrt{2}\,t - gt^2\right)\mathbf{j}\right]$. The ball lands when

$y = 0$ (and $t > 0$) $\Rightarrow$ $t = \dfrac{v_0\sqrt{2}}{g}$ s. Now since it lands 90 m away, $90 = x = \frac{1}{2}v_0\sqrt{2}\,\dfrac{v_0\sqrt{2}}{g}$ or $v_0^2 = 90g$ and the initial velocity is $v_0 = \sqrt{90g} \approx 30$ m/s.

35.
If $\mathbf{r}'(t) = \mathbf{c} \times \mathbf{r}(t)$ then $\mathbf{r}'(t)$ is perpendicular to both $\mathbf{c}$ and $\mathbf{r}(t)$. Remember that $\mathbf{r}'(t)$ points in the direction of motion, so if $\mathbf{r}'(t)$ is always perpendicular to $\mathbf{c}$, the path of the particle must lie in a plane perpendicular to $\mathbf{c}$. But $\mathbf{r}'(t)$ is also perpendicular to the position vector $\mathbf{r}(t)$ which confines the path to a sphere centered at the origin. Considering both restrictions, the path must be contained in a circle that lies in a plane perpendicular to $\mathbf{c}$, and the circle is centered on a line through the origin in the direction of $\mathbf{c}$.

38. $\mathbf{r}(t) = (1+t)\,\mathbf{i} + (t^2 - 2t)\,\mathbf{j} \quad \Rightarrow \quad \mathbf{r}'(t) = \mathbf{i} + (2t-2)\,\mathbf{j}, \quad |\mathbf{r}'(t)| = \sqrt{1^2 + (2t-2)^2} = \sqrt{4t^2 - 8t + 5}$,

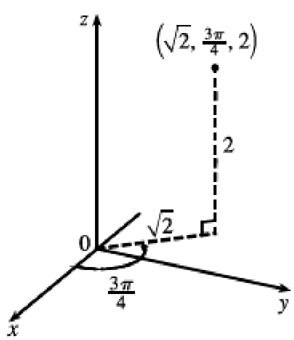
$\mathbf{r}''(t) = 2\,\mathbf{j}, \quad \mathbf{r}'(t) \times \mathbf{r}''(t) = 2\,\mathbf{k}$. Then Equation 9 gives $a_T = \dfrac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|} = \dfrac{2(2t-2)}{\sqrt{4t^2 - 8t + 5}}$ and Equation 10 gives $a_N = \dfrac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|} = \dfrac{2}{\sqrt{4t^2 - 8t + 5}}$.

42. $\mathbf{r}(t) = t\,\mathbf{i} + \cos^2 t\,\mathbf{j} + \sin^2 t\,\mathbf{k} \quad \Rightarrow \quad \mathbf{r}'(t) = \mathbf{i} - 2\cos t \sin t\,\mathbf{j} + 2 \sin t \cos t\,\mathbf{k} = \mathbf{i} - \sin 2t\,\mathbf{j} + \sin 2t\,\mathbf{k}$,

$|\mathbf{r}'(t)| = \sqrt{1 + 2\sin^2 2t}$, $\mathbf{r}''(t) = 2(\sin^2 t - \cos^2 t)\,\mathbf{j} + 2(\cos^2 t - \sin^2 t)\,\mathbf{k} = -2\cos 2t\,\mathbf{j} + 2\cos 2t\,\mathbf{k}$. So

$$a_T = \frac{2\sin 2t \cos 2t + 2 \sin 2t \cos 2t}{\sqrt{1 + 2\sin^2 2t}} = \frac{4 \sin 2t \cos 2t}{\sqrt{1 + 2\sin^2 2t}} \text{ and } a_N = \frac{|-2\cos 2t\,\mathbf{j} - 2\cos 2t\,\mathbf{k}|}{\sqrt{1 + 2\sin^2 2t}} = \frac{2\sqrt{2}\,|\cos 2t|}{\sqrt{1 + 2\sin^2 t}}.$$

2. (a)

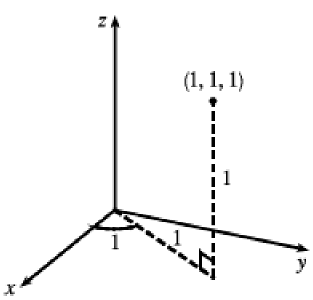


$x = \sqrt{2} \cos \dfrac{3\pi}{4} = \sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -1$,

$y = \sqrt{2} \sin \dfrac{3\pi}{4} = \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = 1$, and $z = 2$,

so the point is $(-1, 1, 2)$ in rectangular coordinates.

(b)



$x = 1 \cos 1 = \cos 1$, $y = 1 \sin 1 = \sin 1$, and $z = 1$,

so the point is $(\cos 1, \sin 1, 1)$ in rectangular coordinates.

4.

(a) $r^2 = \left(2\sqrt{3}\right)^2 + 2^2 = 16$ so $r = 4$; $\tan\theta = \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$ and the point $\left(2\sqrt{3}, 2\right)$ is in the first quadrant of the xy-plane, so $\theta = \frac{\pi}{6} + 2n\pi$; $z = -1$. Thus, one set of cylindrical coordinates is $\left(4, \frac{\pi}{6}, -1\right)$.

(b) $r^2 = 4^2 + (-3)^2 = 25$ so $r = 5$; $\tan\theta = \frac{-3}{4}$ and the point $(4, -3)$ is in the fourth quadrant of the xy-plane, so $\theta = \tan^{-1}\left(-\frac{3}{4}\right) + 2n\pi \approx -0.64 + 2n\pi$; $z = 2$. Thus, one set of cylindrical coordinates is $\left(5, \tan^{-1}\left(-\frac{3}{4}\right) + 2\pi, 2\right) \approx (5, 5.64, 2)$.

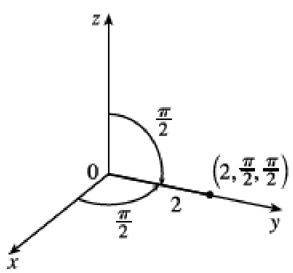
6. Since $r = 5$, $x^2 + y^2 = 25$ and the surface is a circular cylinder with radius 5 and axis the z-axis.

10.

(a) Substituting $x = r\cos\theta$ and $y = r\sin\theta$, the equation $3x + 2y + z = 6$ becomes $3r\cos\theta + 2r\sin\theta + z = 6$ or $z = 6 - r(3\cos\theta + 2\sin\theta)$.

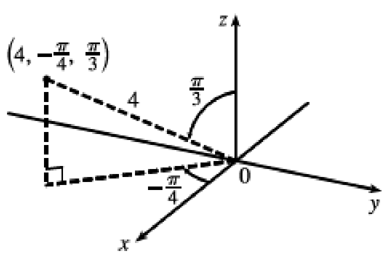
(b) The equation $-x^2 - y^2 + z^2 = 1$ can be written as $-(x^2 + y^2) + z^2 = 1$ which becomes $-r^2 + z^2 = 1$ or $z^2 = 1 + r^2$ in cylindrical coordinates.

2. (a)



$x = 2\sin\frac{\pi}{2}\cos\frac{\pi}{2} = 2\cdot 1\cdot 0 = 0$, $y = 2\sin\frac{\pi}{2}\sin\frac{\pi}{2} = 2\cdot 1\cdot 1 = 2$, $z = 2\cos\frac{\pi}{2} = 2\cdot 0 = 0$ so the point is $(0, 2, 0)$ in rectangular coordinates.

(b)



$x = 4\sin\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) = 4\cdot\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \sqrt{6}$,

$y = 4\sin\frac{\pi}{3}\sin\left(-\frac{\pi}{4}\right) = 4\left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = -\sqrt{6}$,

$z = 4\cos\frac{\pi}{3} = 4\cdot\frac{1}{2} = 2$ so the point is $\left(\sqrt{6}, -\sqrt{6}, 2\right)$ in rectangular coordinates.

4. (a) $\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{1 + 0 + 3} = 2$, $\cos\phi = \dfrac{z}{\rho} = \dfrac{\sqrt{3}}{2} \Rightarrow \phi = \dfrac{\pi}{6}$, and $\cos\theta = \dfrac{x}{\rho\sin\phi} = \dfrac{1}{2\sin(\pi/6)} = 1 \Rightarrow \theta = 0$. Thus spherical coordinates are $\left(2, 0, \dfrac{\pi}{6}\right)$.

(b) $\rho = \sqrt{3 + 1 + 12} = 4$, $\cos\phi = \dfrac{z}{\rho} = \dfrac{2\sqrt{3}}{4} = \dfrac{\sqrt{3}}{2} \Rightarrow \phi = \dfrac{\pi}{6}$, and $\cos\theta = \dfrac{x}{\rho\sin\phi} = \dfrac{\sqrt{3}}{4\sin(\pi/6)} = \dfrac{\sqrt{3}}{2} \Rightarrow \theta = \dfrac{11\pi}{6}$ [since $y < 0$]. Thus spherical coordinates are $\left(4, \dfrac{11\pi}{6}, \dfrac{\pi}{6}\right)$.

7. $\rho = \sin\theta\sin\phi \quad\Rightarrow\quad \rho^2 = \rho\sin\theta\sin\phi \quad\Leftrightarrow\quad x^2+y^2+z^2 = y \quad\Leftrightarrow\quad x^2+y^2-y+\frac14+z^2=\frac14 \quad\Leftrightarrow$

$x^2+\left(y-\frac12\right)^2+z^2=\frac14$. Therefore, the surface is a sphere of radius $\frac12$ centered at $\left(0,\frac12,0\right)$.

8. $\rho^2\left(\sin^2\phi\sin^2\theta+\cos^2\phi\right)=9 \quad\Leftrightarrow\quad (\rho\sin\phi\sin\theta)^2+(\rho\cos\phi)^2=9 \quad\Leftrightarrow\quad y^2+z^2=9$. Thus the surface is a circular cylinder of radius 3 with axis the x-axis.

22. $2x^2-2x+1=0 \quad\Leftrightarrow\quad x=\dfrac{-(-2)\pm\sqrt{(-2)^2-4(2)(1)}}{2(2)}=\dfrac{2\pm\sqrt{-4}}{4}=\dfrac{2\pm2i}{4}=\dfrac12\pm\dfrac12 i$

32. For $z=4\left(\sqrt3+i\right)=4\sqrt3+4i$, $r=\sqrt{\left(4\sqrt3\right)^2+4^2}=\sqrt{64}=8$ and $\tan\theta=\frac{4}{4\sqrt3}=\frac{1}{\sqrt3} \quad\Rightarrow\quad \theta=\frac{\pi}{6} \quad\Rightarrow$

$z=8\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$. For $w=-3-3i$, $r=\sqrt{(-3)^2+(-3)^2}=\sqrt{18}=3\sqrt2$ and $\tan\theta=\frac{-3}{-3}=1 \quad\Rightarrow\quad \theta=\frac{5\pi}{4} \quad\Rightarrow$

$w=3\sqrt2\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$. Therefore, $zw=8\cdot3\sqrt2\left[\cos\left(\frac{\pi}{6}+\frac{5\pi}{4}\right)+i\sin\left(\frac{\pi}{6}+\frac{5\pi}{4}\right)\right]=24\sqrt2\left(\cos\frac{17\pi}{12}+i\sin\frac{17\pi}{12}\right)$,

$z/w=\frac{8}{3\sqrt2}\left[\cos\left(\frac{\pi}{6}-\frac{5\pi}{4}\right)+i\sin\left(\frac{\pi}{6}-\frac{5\pi}{4}\right)\right]=\frac{4\sqrt2}{3}\left[\cos\left(-\frac{13\pi}{12}\right)+i\sin\left(-\frac{13\pi}{12}\right)\right]$, and $1/z=\frac18\left(\cos\frac{\pi}{6}-i\sin\frac{\pi}{6}\right)$.

36. For $z=1-i$, $r=\sqrt2$ and $\tan\theta=\frac{-1}{1}=-1 \quad\Rightarrow\quad \theta=\frac{7\pi}{4} \quad\Rightarrow\quad z=\sqrt2\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right) \quad\Rightarrow$

$(1-i)^8=\left[\sqrt2\left(\cos\frac{7\pi}{4}+i\sin\frac{7\pi}{4}\right)\right]^8=2^4\left(\cos\frac{8\cdot7\pi}{4}+i\sin\frac{8\cdot7\pi}{4}\right)=16(\cos14\pi+i\sin14\pi)=16(1+0i)=16$.

38. $32=32+0i=32(\cos0+i\sin0)$. Using Equation 3 with $r=32$, $n=5$, and $\theta=0$, we have

$w_k=32^{1/5}\left[\cos\left(\dfrac{0+2k\pi}{5}\right)+i\sin\left(\dfrac{0+2k\pi}{5}\right)\right]=2\left(\cos\frac25\pi k+i\sin\frac25\pi k\right)$, where $k=0,1,2,3,4$.

$w_0=2(\cos0+i\sin0)=2$

$w_1=2\left(\cos\frac{2\pi}{5}+i\sin\frac{2\pi}{5}\right)$

$w_2=2\left(\cos\frac{4\pi}{5}+i\sin\frac{4\pi}{5}\right)$

$w_3=2\left(\cos\frac{6\pi}{5}+i\sin\frac{6\pi}{5}\right)$

$w_4=2\left(\cos\frac{8\pi}{5}+i\sin\frac{8\pi}{5}\right)$