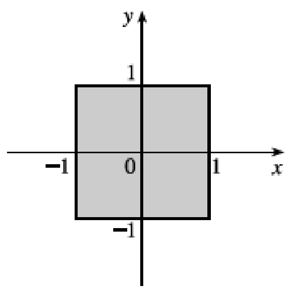
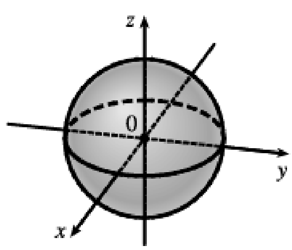
12. (a) $g(1,2,3) = 1^3 \cdot 2^2 \cdot 3\sqrt{10-1-2-3} = 12\sqrt{4} = 24$

(b) g is defined only when $10 - x - y - z \geq 0 \quad \Leftrightarrow \quad z \leq 10 - x - y$, so the domain is $\{(x,y,z) \mid z \leq 10 - x - y\}$, the points on or below the plane $x + y + z = 10$.

17. $\sqrt{1-x^2}$ is defined only when $1 - x^2 \geq 0$, or $x^2 \leq 1 \quad \Leftrightarrow \quad -1 \leq x \leq 1$, and $\sqrt{1-y^2}$ is defined only when $1 - y^2 \geq 0$, or $y^2 \leq 1 \quad \Leftrightarrow \quad -1 \leq y \leq 1$. Thus the domain of f is $\{(x,y) \mid -1 \leq x \leq 1,\ -1 \leq y \leq 1\}$.



21. We need $1 - x^2 - y^2 - z^2 \geq 0$ or $x^2 + y^2 + z^2 \leq 1$, so $D = \{(x,y,z) \mid x^2 + y^2 + z^2 \leq 1\}$ (the points inside or on the sphere of radius 1, center the origin).

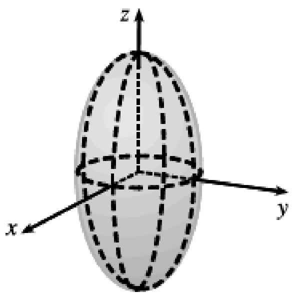


22. f is defined only when $16 - 4x^2 - 4y^2 - z^2 > 0 \quad \Rightarrow$

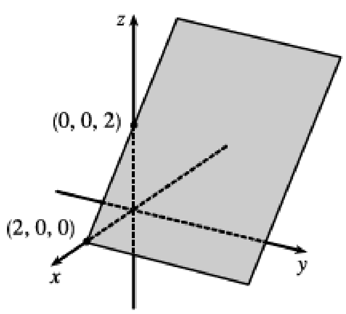
$\dfrac{x^2}{4} + \dfrac{y^2}{4} + \dfrac{z^2}{16} < 1$. Thus,

$D = \left\{ (x, y, z) \,\middle|\, \dfrac{x^2}{4} + \dfrac{y^2}{4} + \dfrac{z^2}{16} < 1 \right\}$, that is, the points

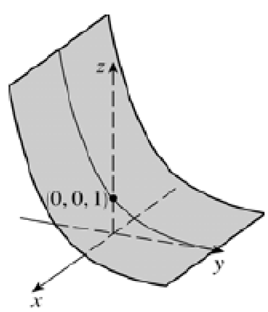
inside the ellipsoid $\dfrac{x^2}{4} + \dfrac{y^2}{4} + \dfrac{z^2}{16} = 1$.



24. $z = 2 - x$, a plane which intersects the xz-plane in the line $z = 2 - x$, $y = 0$. The portion of this plane for $y \geq 0$, $z \geq 0$ is shown.



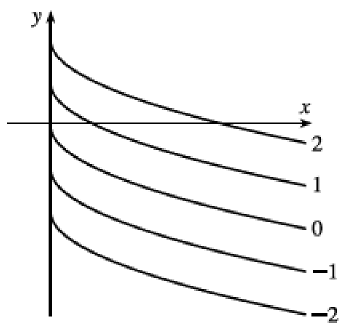
26. $z = e^{-y}$, a cylinder.



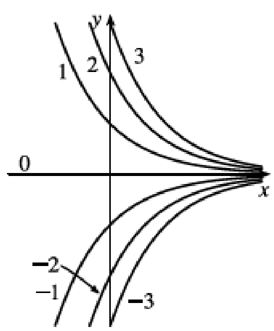
36.

If we start at the origin and move along the x-axis, for example, the z-values of a cone centered at the origin increase at a constant rate, so we would expect its level curves to be equally spaced. A paraboloid with vertex the origin, on the other hand, has z-values which change slowly near the origin and more quickly as we move farther away. Thus, we would expect its level curves near the origin to be spaced more widely apart than those farther from the origin. Therefore contour map I must correspond to the paraboloid, and contour map II the cone.

45. The level curves are $\sqrt{x} + y = k$ or $y = -\sqrt{x} + k$, a family of vertical translations of the graph of the root function $y = -\sqrt{x}$.



47. The level curves are $ye^x = k$ or $y = ke^{-x}$, a family of exponential curves.



59.

$z = \sin(xy)$ (a) C (b) II

Reasons: This function is periodic in both x and y, and the function is the same when x is interchanged with y, so its graph is symmetric about the plane $y = x$. In addition, the function is 0 along the x- and y-axes. These conditions are satisfied only by C and II.

60. $z = e^x \cos y$ (a) A (b) IV

Reasons: This function is periodic in y but not x, a condition satisfied only by A and IV. Also, note that traces in $x = k$ are cosine curves with amplitude that increases as x increases.

61. $z = \sin(x - y)$ (a) F (b) I

Reasons: This function is periodic in both x and y but is constant along the lines $y = x + k$, a condition satisfied only by F and I.

62.
$z = \sin x - \sin y$ (a) E (b) III

Reasons: This function is periodic in both x and y, but unlike the function in Exercise 61, it is not constant along lines such as $y = x + \pi$, so the contour map is III. Also notice that traces in $y = k$ are vertically shifted copies of the sine wave $z = \sin x$, so the graph must be E.

63. $z = (1 - x^2)(1 - y^2)$ (a) B (b) VI

Reasons: This function is 0 along the lines $x = \pm 1$ and $y = \pm 1$. The only contour map in which this could occur is VI. Also note that the trace in the xz-plane is the parabola $z = 1 - x^2$ and the trace in the yz-plane is the parabola $z = 1 - y^2$, so the graph is B.

64. $z = \dfrac{x - y}{1 + x^2 + y^2}$ (a) D (b) V

Reasons: This function is not periodic, ruling out the graphs in A, C, E, and F. Also, the values of z approach 0 as we use points farther from the origin. The only graph that shows this behavior is D, which corresponds to V.

5. $f(x, y) = 5x^3 - x^2y^2$ is a polynomial, and hence continuous, so $\lim_{(x,y)\to(1,2)} f(x, y) = f(1, 2) = 5(1)^3 - (1)^2(2)^2 = 1$.

6. $-xy$ is a polynomial and therefore continuous. Since e^t is a continuous function, the composition e^{-xy} is also continuous. Similarly, $x + y$ is a polynomial and $\cos t$ is a continuous function, so the composition $\cos(x + y)$ is continuous. The product of continuous functions is continuous, so $f(x, y) = e^{-xy}\cos(x + y)$ is a continuous function and

$$\lim_{(x,y)\to(1,-1)} f(x, y) = f(1, -1) = e^{-(1)(-1)}\cos(1 + (-1)) = e^1 \cos 0 = e.$$

8. $\dfrac{1+y^2}{x^2+xy}$ is a rational function and hence continuous on its domain, which includes $(1,0)$. $\ln t$ is a continuous function for $t>0$, so the composition $f(x,y)=\ln\left(\dfrac{1+y^2}{x^2+xy}\right)$ is continuous wherever $\dfrac{1+y^2}{x^2+xy}>0$. In particular, f is continuous at $(1,0)$ and so $\lim\limits_{(x,y)\to(1,0)} f(x,y)=f(1,0)=\ln\left(\dfrac{1+0^2}{1^2+1\cdot 0}\right)=\ln\dfrac{1}{1}=0$.

11. $f(x,y)=(y^2\sin^2 x)/(x^4+y^4)$. On the x-axis, $f(x,0)=0$ for $x\neq 0$, so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along the x-axis. Approaching $(0,0)$ along the line $y=x$, $f(x,x)=\dfrac{x^2\sin^2 x}{x^4+x^4}=\dfrac{\sin^2 x}{2x^2}=\dfrac{1}{2}\left(\dfrac{\sin x}{x}\right)^2$ for $x\neq 0$ and $\lim\limits_{x\to 0}\dfrac{\sin x}{x}=1$, so $f(x,y)\to\frac{1}{2}$. Since f has two different limits along two different lines, the limit does not exist.

12. $f(x,y)=\dfrac{xy-y}{(x-1)^2+y^2}$. On the x-axis, $f(x,0)=0/(x-1)^2=0$ for $x\neq 1$, so $f(x,y)\to 0$ as $(x,y)\to(1,0)$ along the x-axis. Approaching $(1,0)$ along the line $y=x-1$, $f(x,x-1)=\dfrac{x(x-1)-(x-1)}{(x-1)^2+(x-1)^2}=\dfrac{(x-1)^2}{2(x-1)^2}=\dfrac{1}{2}$ for $x\neq 1$, so $f(x,y)\to\frac{1}{2}$ along this line. Thus the limit does not exist.

13. $f(x,y)=\dfrac{xy}{\sqrt{x^2+y^2}}$. We can see that the limit along any line through $(0,0)$ is 0, as well as along other paths through $(0,0)$ such as $x=y^2$ and $y=x^2$. So we suspect that the limit exists and equals 0; we use the Squeeze Theorem to prove our assertion. $0\leq\left|\dfrac{xy}{\sqrt{x^2+y^2}}\right|\leq|x|$ since $|y|\leq\sqrt{x^2+y^2}$, and $|x|\to 0$ as $(x,y)\to(0,0)$. So $\lim\limits_{(x,y)\to(0,0)} f(x,y)=0$.

18. $f(x,y)=xy^4/(x^2+y^8)$. On the x-axis, $f(x,0)=0$ for $x\neq 0$, so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along the x-axis. Approaching $(0,0)$ along the curve $x=y^4$ gives $f(y^4,y)=y^8/2y^8=\frac{1}{2}$ for $y\neq 0$, so along this path $f(x,y)\to\frac{1}{2}$ as $(x,y)\to(0,0)$. Thus the limit does not exist.

19. e^{y^2} is a composition of continuous functions and hence continuous. xz is a continuous function and $\tan t$ is continuous for $t\neq\frac{\pi}{2}+n\pi$ (n an integer), so the composition $\tan(xz)$ is continuous for $xz\neq\frac{\pi}{2}+n\pi$. Thus the product $f(x,y,z)=e^{y^2}\tan(xz)$ is a continuous function for $xz\neq\frac{\pi}{2}+n\pi$. If $x=\pi$ and $z=\frac{1}{3}$ then $xz\neq\frac{\pi}{2}+n\pi$, so $\lim\limits_{(x,y,z)\to(\pi,0,1/3)} f(x,y,z)=f(\pi,0,1/3)=e^{0^2}\tan(\pi\cdot 1/3)=1\cdot\tan(\pi/3)=\sqrt{3}$.

37. $f(x,y)=\begin{cases}\dfrac{x^2y^3}{2x^2+y^2} & \text{if } (x,y)\neq(0,0)\\ 1 & \text{if } (x,y)=(0,0)\end{cases}$ The first piece of f is a rational function defined everywhere except at the origin, so f is continuous on $\mathbb{R}^2$ except possibly at the origin. Since $x^2 \le 2x^2+y^2$, we have $\left|x^2y^3/(2x^2+y^2)\right| \le \left|y^3\right|$. We know that $\left|y^3\right| \to 0$ as $(x,y)\to(0,0)$. So, by the Squeeze Theorem, $\lim\limits_{(x,y)\to(0,0)} f(x,y) = \lim\limits_{(x,y)\to(0,0)} \dfrac{x^2y^3}{2x^2+y^2} = 0$. But $f(0,0)=1$, so f is discontinuous at $(0,0)$. Therefore, f is continuous on the set $\{(x,y)\mid(x,y)\neq(0,0)\}$.

41. $$\lim_{(x,y)\to(0,0)} \frac{e^{-x^2-y^2}-1}{x^2+y^2} = \lim_{r\to0+} \frac{e^{-r^2}-1}{r^2} = \lim_{r\to0+} \frac{e^{-r^2}(-2r)}{2r} \quad \text{[using l'Hospital's Rule]}$$
$$= \lim_{r\to0+} -e^{-r^2} = -e^0 = -1$$

26. $u(r,\theta)=\sin(r\cos\theta) \quad\Rightarrow\quad u_r(r,\theta)=\cos(r\cos\theta)\cdot\cos\theta=\cos\theta\cos(r\cos\theta)$,

$u_\theta(r,\theta)=\cos(r\cos\theta)(-r\sin\theta)=-r\sin\theta\cos(r\cos\theta)$

28. $f(x,y)=x^y \quad\Rightarrow\quad f_x(x,y)=yx^{y-1},\; f_y(x,y)=x^y\ln x$

30. $F(\alpha,\beta)=\displaystyle\int_\alpha^\beta \sqrt{t^3+1}\,dt \quad\Rightarrow$

$F_\alpha(\alpha,\beta)=\dfrac{\partial}{\partial\alpha}\displaystyle\int_\alpha^\beta\sqrt{t^3+1}\,dt=\dfrac{\partial}{\partial\alpha}\left[-\int_\beta^\alpha\sqrt{t^3+1}\,dt\right]=-\dfrac{\partial}{\partial\alpha}\int_\beta^\alpha\sqrt{t^3+1}\,dt=-\sqrt{\alpha^3+1}$ by the Fundamental Theorem of Calculus, Part 1; $\quad F_\beta(\alpha,\beta)=\dfrac{\partial}{\partial\beta}\displaystyle\int_\alpha^\beta\sqrt{t^3+1}\,dt=\sqrt{\beta^3+1}$.

39. $u=\sqrt{x_1^2+x_2^2+\cdots+x_n^2}$. For each $i=1,\ldots,n$, $u_{x_i}=\frac12\left(x_1^2+x_2^2+\cdots+x_n^2\right)^{-1/2}(2x_i)=\dfrac{x_i}{\sqrt{x_1^2+x_2^2+\cdots+x_n^2}}$.

40. $u=\sin(x_1+2x_2+\cdots+nx_n)$. For each $i=1,\ldots,n$, $u_{x_i}=i\cos(x_1+2x_2+\cdots+nx_n)$.

49. $e^z=xyz \quad\Rightarrow\quad \dfrac{\partial}{\partial x}(e^z)=\dfrac{\partial}{\partial x}(xyz) \quad\Rightarrow\quad e^z\dfrac{\partial z}{\partial x}=y\left(x\dfrac{\partial z}{\partial x}+z\cdot1\right) \quad\Rightarrow\quad e^z\dfrac{\partial z}{\partial x}-xy\dfrac{\partial z}{\partial x}=yz \quad\Rightarrow$

$(e^z-xy)\dfrac{\partial z}{\partial x}=yz$, so $\dfrac{\partial z}{\partial x}=\dfrac{yz}{e^z-xy}$.

$\dfrac{\partial}{\partial y}(e^z)=\dfrac{\partial}{\partial y}(xyz) \quad\Rightarrow\quad e^z\dfrac{\partial z}{\partial y}=x\left(y\dfrac{\partial z}{\partial x}+z\cdot1\right) \quad\Rightarrow\quad e^z\dfrac{\partial z}{\partial y}-xy\dfrac{\partial z}{\partial y}=xz \quad\Rightarrow\quad (e^z-xy)\dfrac{\partial z}{\partial y}=xz$, so

$\dfrac{\partial z}{\partial y}=\dfrac{xz}{e^z-xy}$.

50. $yz + x\ln y = z^2 \quad\Rightarrow\quad \dfrac{\partial}{\partial x}(yz + x\ln y) = \dfrac{\partial}{\partial x}(z^2) \quad\Rightarrow\quad y\,\dfrac{\partial z}{\partial x} + \ln y = 2z\,\dfrac{\partial z}{\partial x} \quad\Rightarrow\quad \ln y = 2z\,\dfrac{\partial z}{\partial x} - y\,\dfrac{\partial z}{\partial x} \quad\Rightarrow$

$\ln y = (2z - y)\,\dfrac{\partial z}{\partial x}$, so $\dfrac{\partial z}{\partial x} = \dfrac{\ln y}{2z - y}$.

$\dfrac{\partial}{\partial y}(yz + x\ln y) = \dfrac{\partial}{\partial y}(z^2) \quad\Rightarrow\quad y\,\dfrac{\partial z}{\partial y} + z\cdot 1 + x\cdot\dfrac{1}{y} = 2z\,\dfrac{\partial z}{\partial y} \quad\Rightarrow\quad z + \dfrac{x}{y} = 2z\,\dfrac{\partial z}{\partial y} - y\,\dfrac{\partial z}{\partial y} \quad\Rightarrow$

$z + \dfrac{x}{y} = (2z - y)\,\dfrac{\partial z}{\partial y}$, so $\dfrac{\partial z}{\partial y} = \dfrac{z + (x/y)}{2z - y} = \dfrac{x + yz}{y(2z - y)}$.

51. (a) $z = f(x) + g(y) \quad\Rightarrow\quad \dfrac{\partial z}{\partial x} = f'(x), \quad \dfrac{\partial z}{\partial y} = g'(y)$

(b) $z = f(x + y)$. Let $u = x + y$. Then $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = \dfrac{df}{du}(1) = f'(u) = f'(x + y)$,

$\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = \dfrac{df}{du}(1) = f'(u) = f'(x + y)$.

52. (a) $z = f(x)g(y) \quad\Rightarrow\quad \dfrac{\partial z}{\partial x} = f'(x)g(y), \quad \dfrac{\partial z}{\partial y} = f(x)g'(y)$

(b) $z = f(xy)$. Let $u = xy$. Then $\dfrac{\partial u}{\partial x} = y$ and $\dfrac{\partial u}{\partial y} = x$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = \dfrac{df}{du}\cdot y = yf'(u) = yf'(xy)$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = \dfrac{df}{du}\cdot x = xf'(u) = xf'(xy)$.

(c) $z = f\left(\dfrac{x}{y}\right)$. Let $u = \dfrac{x}{y}$. Then $\dfrac{\partial u}{\partial x} = \dfrac{1}{y}$ and $\dfrac{\partial u}{\partial y} = -\dfrac{x}{y^2}$. Hence $\dfrac{\partial z}{\partial x} = \dfrac{df}{du}\dfrac{\partial u}{\partial x} = f'(u)\dfrac{1}{y} = \dfrac{f'(x/y)}{y}$

and $\dfrac{\partial z}{\partial y} = \dfrac{df}{du}\dfrac{\partial u}{\partial y} = f'(u)\left(-\dfrac{x}{y^2}\right) = -\dfrac{xf'(x/y)}{y^2}$.

65. $f(x,y,z) = e^{xyz^2} \quad\Rightarrow\quad f_x = e^{xyz^2}\cdot yz^2 = yz^2e^{xyz^2}$, $f_{xy} = yz^2\cdot e^{xyz^2}(xz^2) + e^{xyz^2}\cdot z^2 = (xyz^4 + z^2)e^{xyz^2}$,

$f_{xyz} = (xyz^4 + z^2)\cdot e^{xyz^2}(2xyz) + e^{xyz^2}\cdot(4xyz^3 + 2z) = (2x^2y^2z^5 + 6xyz^3 + 2z)e^{xyz^2}$.

72. Let $f(x,y,z) = \sqrt{1 + xz}$ and $h(x,y,z) = \sqrt{1 - xy}$ so that $g = f + h$. Then $f_y = 0 = f_{yx} = f_{yxz}$ and $h_z = 0 = h_{zx} = h_{zxy}$. But (since the partial derivatives are continous on their domains) $f_{xyz} = f_{yxz}$ and $h_{xyz} = h_{zxy}$, so $g_{xyz} = f_{xyz} + h_{xyz} = 0 + 0 = 0$.

2. $z = f(x,y) = 3(x-1)^2 + 2(y+3)^2 + 7 \quad\Rightarrow\quad f_x(x,y) = 6(x-1)$, $f_y(x,y) = 4(y+3)$, so $f_x(2,-2) = 6$ and $f_y(2,-2) = 4$. By Equation 2, an equation of the tangent plane is $z - 12 = f_x(2,-2)(x-2) + f_y(2,-2)\,[y - (-2)] \quad\Rightarrow$

$z - 12 = 6(x-2) + 4(y+2)$ or $z = 6x + 4y + 8$.

4. $z = f(x,y) = xe^{xy} \quad \Rightarrow \quad f_x(x,y) = xye^{xy} + e^{xy},\ f_y(x,y) = x^2e^{xy},$ so $f_x(2,0) = 1,\ f_y(2,0) = 4$, and an equation of the tangent plane is $z - 2 = f_x(2,0)(x-2) + f_y(2,0)(y-0) \quad \Rightarrow \quad z - 2 = 1(x-2) + 4(y-0)$ or $z = x + 4y$.

13. $f(x,y) = \dfrac{x}{x+y}$. The partial derivatives are $f_x(x,y) = \dfrac{1(x+y) - x(1)}{(x+y)^2} = y/(x+y)^2$ and

$f_y(x,y) = x(-1)(x+y)^{-2} \cdot 1 = -x/(x+y)^2$, so $f_x(2,1) = \frac{1}{9}$ and $f_y(2,1) = -\frac{2}{9}$. Both f_x and f_y are continuous functions for $y \neq -x$, so f is differentiable at $(2,1)$ by Theorem 8. The linearization of f at $(2,1)$ is given by

$L(x,y) = f(2,1) + f_x(2,1)(x-2) + f_y(2,1)(y-1) = \frac{2}{3} + \frac{1}{9}(x-2) - \frac{2}{9}(y-1) = \frac{1}{9}x - \frac{2}{9}y + \frac{2}{3}$.

15. $f(x,y) = e^{-xy}\cos y$. The partial derivatives are $f_x(x,y) = e^{-xy}(-y)\cos y = -ye^{-xy}\cos y$ and

$f_y(x,y) = e^{-xy}(-\sin y) + (\cos y)e^{-xy}(-x) = -e^{-xy}(\sin y + x\cos y)$, so $f_x(\pi,0) = 0$ and $f_y(\pi,0) = -\pi$.

Both f_x and f_y are continuous functions, so f is differentiable at $(\pi,0)$, and the linearization of f at $(\pi,0)$ is

$L(x,y) = f(\pi,0) + f_x(\pi,0)(x-\pi) + f_y(\pi,0)(y-0) = 1 + 0(x-\pi) - \pi(y-0) = 1 - \pi y$.

18. Let $f(x,y) = \sqrt{y + \cos^2 x}$. Then $f_x(x,y) = \frac{1}{2}(y + \cos^2 x)^{-1/2}(2\cos x)(-\sin x) = -\cos x \sin x/\sqrt{y + \cos^2 x}$ and

$f_y(x,y) = \frac{1}{2}(y + \cos^2 x)^{-1/2}(1) = 1/\left(2\sqrt{y + \cos^2 x}\right)$. Both f_x and f_y are continuous functions for $y > -\cos^2 x$, so f is differentiable at $(0,0)$ by Theorem 8. We have $f_x(0,0) = 0$ and $f_y(0,0) = \frac{1}{2}$, so the linear approximation of f at $(0,0)$ is

$f(x,y) \approx f(0,0) + f_x(0,0)(x-0) + f_y(0,0)(y-0) = 1 + 0x + \frac{1}{2}y = 1 + \frac{1}{2}y$.

19. We can estimate $f(2.2, 4.9)$ using a linear approximation of f at $(2,5)$, given by

$f(x,y) \approx f(2,5) + f_x(2,5)(x-2) + f_y(2,5)(y-5) = 6 + 1(x-2) + (-1)(y-5) = x - y + 9$. Thus

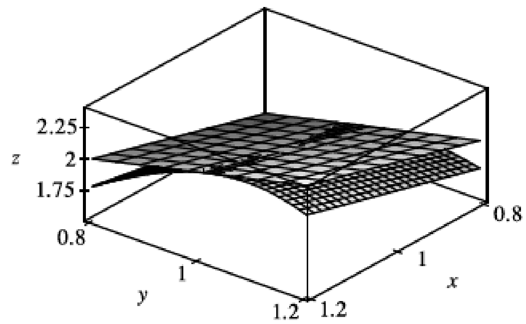
$f(2.2, 4.9) \approx 2.2 - 4.9 + 9 = 6.3$.

20. $f(x,y) = 1 - xy\cos\pi y \quad \Rightarrow \quad f_x(x,y) = -y\cos\pi y$ and

$f_y(x,y) = -x[y(-\pi\sin\pi y) + (\cos\pi y)(1)] = \pi xy\sin\pi y - x\cos\pi y$, so $f_x(1,1) = 1$, $f_y(1,1) = 1$. Then the linear approximation of f at $(1,1)$ is given by

$$\begin{aligned} f(x,y) &\approx f(1,1) + f_x(1,1)(x-1) + f_y(1,1)(y-1) \\ &= 2 + (1)(x-1) + (1)(y-1) = x + y \end{aligned}$$

Thus $f(1.02, 0.97) \approx 1.02 + 0.97 = 1.99$. We graph f and its tangent plane near the point $(1,1,2)$ below. Notice near $y = 1$ the surfaces are almost identical.



26. $u = \sqrt{x^2 + 3y^2} = (x^2 + 3y^2)^{1/2} \quad \Rightarrow$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = \tfrac{1}{2}(x^2+3y^2)^{-1/2}(2x)\,dx + \tfrac{1}{2}(x^2+3y^2)^{-1/2}(6y)\,dy = \frac{x}{\sqrt{x^2+3y^2}}\,dx + \frac{3y}{\sqrt{x^2+3y^2}}\,dy$$

27. $m = p^5q^3 \quad \Rightarrow \quad dm = \dfrac{\partial m}{\partial p}\,dp + \dfrac{\partial m}{\partial q}\,dq = 5p^4q^3\,dp + 3p^5q^2\,dq$

32. $dx = \Delta x = -0.04$, $dy = \Delta y = 0.05$, $z = x^2 - xy + 3y^2$, $z_x = 2x - y$, $z_y = 6y - x$. Thus when $x = 3$ and $y = -1$,

$dz = (7)(-0.04) + (-9)(0.05) = -0.73$ while $\Delta z = (2.96)^2 - (2.96)(-0.95) + 3(-0.95)^2 - (9 + 3 + 3) = -0.7189$.

34. Let V be the volume. Then $V = \pi r^2 h$ and $\Delta V \approx dV = 2\pi rh\,dr + \pi r^2\,dh$ is an estimate of the amount of metal. With $dr = 0.05$ and $dh = 0.2$ we get $dV = 2\pi(2)(10)(0.05) + \pi(2)^2(0.2) = 2.80\pi \approx 8.8\text{ cm}^3$.