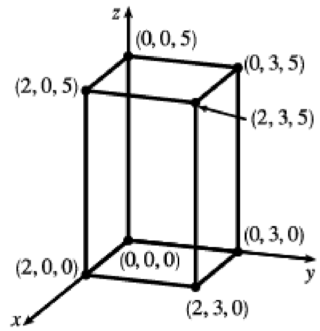
4. The projection of $(2, 3, 5)$ onto the xy-plane is $(2, 3, 0)$;

onto the yz-plane, $(0, 3, 5)$; onto the xz-plane, $(2, 0, 5)$.

The length of the diagonal of the box is the distance between the origin and $(2, 3, 5)$, given by

$$\sqrt{(2-0)^2+(3-0)^2+(5-0)^2}=\sqrt{38}\approx 6.16$$



12.
An equation of the sphere with center $(2, -6, 4)$ and radius 5 is $(x-2)^2+[y-(-6)]^2+(z-4)^2=5^2$ or $(x-2)^2+(y+6)^2+(z-4)^2=25$. The intersection of this sphere with the xy-plane is the set of points on the sphere whose z-coordinate is 0. Putting $z=0$ into the equation, we have $(x-2)^2+(y+6)^2=9$, $z=0$ which represents a circle in the xy-plane with center $(2, -6, 0)$ and radius 3. To find the intersection with the xz-plane, we set $y=0$: $(x-2)^2+(z-4)^2=-11$. Since no points satisfy this equation, the sphere does not intersect the xz-plane. (Also note that the distance from the center of the sphere to the xz-plane is greater than the radius of the sphere.) To find the intersection with the yz-plane, we set $x=0$: $(y+6)^2+(z-4)^2=21$, $x=0$, a circle in the yz-plane with center $(0, -6, 4)$ and radius $\sqrt{21}$.

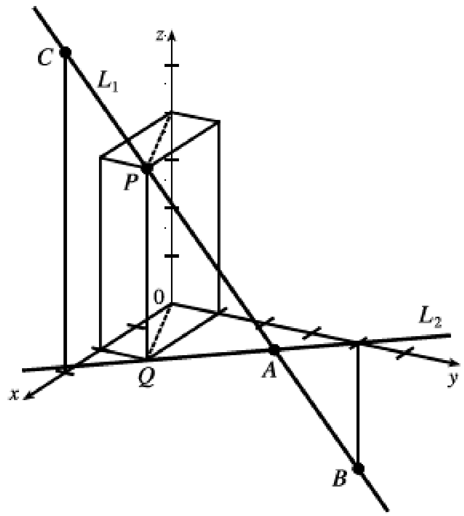
15.
Completing squares in the equation $x^2+y^2+z^2-2x-4y+8z=15$ gives

$(x^2-2x+1)+(y^2-4y+4)+(z^2+8z+16)=15+1+4+16 \quad\Rightarrow\quad (x-1)^2+(y-2)^2+(z+4)^2=36$, which we recognize as an equation of a sphere with center $(1, 2, -4)$ and radius 6.

37. This describes a region all of whose points have a distance to the origin which is greater than r, but smaller than R. So inequalities describing the region are $r<\sqrt{x^2+y^2+z^2}<R$, or $r^2<x^2+y^2+z^2<R^2$.

39. (a) To find the x- and y-coordinates of the point P, we project it onto L_2 and project the resulting point Q onto the x- and y-axes. To find the z-coordinate, we project P onto either the xz-plane or the yz-plane (using our knowledge of its x- or y-coordinate) and then project the resulting point onto the z-axis. (Or, we could draw a line parallel to QO from P to the z-axis.) The coordinates of P are $(2, 1, 4)$.

(b) A is the intersection of L_1 and L_2, B is directly below the y-intercept of L_2, and C is directly above the x-intercept of L_2.



3. Vectors are equal when they share the same length and direction (but not necessarily location). Using the symmetry of the parallelogram as a guide, we see that $\overrightarrow{AB} = \overrightarrow{DC}$, $\overrightarrow{DA} = \overrightarrow{CB}$, $\overrightarrow{DE} = \overrightarrow{EB}$, and $\overrightarrow{EA} = \overrightarrow{CE}$.

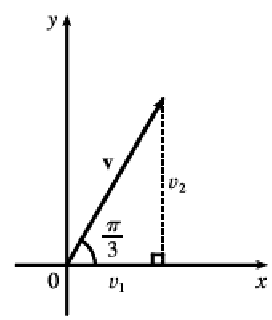
21. $\mathbf{a} + \mathbf{b} = (\mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}) + (-2\,\mathbf{i} - \mathbf{j} + 5\,\mathbf{k}) = -\,\mathbf{i} + \mathbf{j} + 2\mathbf{k}$

$2\mathbf{a} + 3\mathbf{b} = 2\,(\mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}) + 3\,(-2\,\mathbf{i} - \mathbf{j} + 5\,\mathbf{k}) = 2\,\mathbf{i} + 4\,\mathbf{j} - 6\,\mathbf{k} - 6\,\mathbf{i} - 3\,\mathbf{j} + 15\,\mathbf{k} = -\,4\,\mathbf{i} + \mathbf{j} + 9\mathbf{k}$

$|\mathbf{a}| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14}$

$|\mathbf{a} - \mathbf{b}| = |(\mathbf{i} + 2\,\mathbf{j} - 3\,\mathbf{k}) - (-2\,\mathbf{i} - \mathbf{j} + 5\,\mathbf{k})| = |3\,\mathbf{i} + 3\,\mathbf{j} - 8\,\mathbf{k}| = \sqrt{3^2 + 3^2 + (-8)^2} = \sqrt{82}$

29. From the figure, we see that the x-component of $\mathbf{v}$ is
$v_1 = |\mathbf{v}|\cos(\pi/3) = 4 \cdot \frac{1}{2} = 2$ and the y-component is
$v_2 = |\mathbf{v}|\sin(\pi/3) = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}$. Thus
$\mathbf{v} = \langle v_1, v_2 \rangle = \langle 2, 2\sqrt{3}\,\rangle$.



32. The given force vectors can be expressed in terms of their horizontal and vertical components as $20\cos 45^\circ\,\mathbf{i} + 20\sin 45^\circ\,\mathbf{j} = 10\sqrt{2}\,\mathbf{i} + 10\sqrt{2}\,\mathbf{j}$ and $16\cos 30^\circ\,\mathbf{i} - 16\sin 30^\circ\,\mathbf{j} = 8\sqrt{3}\,\mathbf{i} - 8\,\mathbf{j}$. The resultant force $\mathbf{F}$ is the sum of these two vectors: $\mathbf{F} = \left(10\sqrt{2} + 8\sqrt{3}\right)\mathbf{i} + \left(10\sqrt{2} - 8\right)\mathbf{j} \approx 28.00\,\mathbf{i} + 6.14\,\mathbf{j}$. Then we have $|\mathbf{F}| \approx \sqrt{(28.00)^2 + (6.14)^2} \approx 28.7$ lb and, letting θ be the angle $\mathbf{F}$ makes with the positive x-axis,

$$\tan\theta = \frac{10\sqrt{2} - 8}{10\sqrt{2} + 8\sqrt{3}} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{10\sqrt{2} - 8}{10\sqrt{2} + 8\sqrt{3}}\right) \approx 12.4^\circ.$$

51. Consider triangle ABC, where D and E are the midpoints of AB and BC. We know that $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$ $\quad$ (**1**) and $\overrightarrow{DB} + \overrightarrow{BE} = \overrightarrow{DE}$ $\quad$ (**2**). However, $\overrightarrow{DB} = \frac{1}{2}\overrightarrow{AB}$, and $\overrightarrow{BE} = \frac{1}{2}\overrightarrow{BC}$. Substituting these expressions for $\overrightarrow{DB}$ and $\overrightarrow{BE}$ into (**2**) gives $\frac{1}{2}\overrightarrow{AB} + \frac{1}{2}\overrightarrow{BC} = \overrightarrow{DE}$. Comparing this with (**1**) gives $\overrightarrow{DE} = \frac{1}{2}\overrightarrow{AC}$. Therefore $\overrightarrow{AC}$ and $\overrightarrow{DE}$ are parallel and $\left|\overrightarrow{DE}\right| = \frac{1}{2}\left|\overrightarrow{AC}\right|$.

5. Since $\theta = \frac{\pi}{4}$ but r and z may vary, the surface is a vertical half-plane including the z-axis and intersecting the xy-plane in the half-line $y = x$, $x \geq 0$.

7. $z = 4 - r^2 = 4 - (x^2 + y^2)$ or $4 - x^2 - y^2$, so the surface is a circular paraboloid with vertex $(0, 0, 4)$, axis the z-axis, and opening downward.

9. (a) Substituting $x^2 + y^2 = r^2$ and $x = r\cos\theta$, the equation $x^2 - x + y^2 + z^2 = 1$ becomes $r^2 - r\cos\theta + z^2 = 1$ or $z^2 = 1 + r\cos\theta - r^2$.

(b) Substituting $x = r\cos\theta$ and $y = r\sin\theta$, the equation $z = x^2 - y^2$ becomes $z = (r\cos\theta)^2 - (r\sin\theta)^2 = r^2(\cos^2\theta - \sin^2\theta)$ or $z = r^2\cos 2\theta$.

5. Since $\phi = \frac{\pi}{3}$, the surface is the top half of the right circular cone with vertex at the origin and axis the positive z-axis.

6. Since $\rho = 3$, $x^2 + y^2 + z^2 = 9$ and the surface is a sphere with center the origin and radius 3.

10. (a) $x^2 - 2x + y^2 + z^2 = 0 \quad \Leftrightarrow \quad (x^2 + y^2 + z^2) - 2x = 0 \quad \Leftrightarrow \quad \rho^2 - 2(\rho\sin\phi\cos\theta) = 0$ or $\rho = 2\sin\phi\cos\theta$.

(b) $x + 2y + 3z = 1 \quad \Leftrightarrow \quad \rho\sin\phi\cos\theta + 2\rho\sin\phi\sin\theta + 3\rho\cos\phi = 1$ or $\rho = 1/(\sin\phi\cos\theta + 2\sin\phi\sin\theta + 3\cos\phi)$.