1. (a) $\mathbf{a}\cdot\mathbf{b}$ is a scalar, and the dot product is defined only for vectors, so $(\mathbf{a}\cdot\mathbf{b})\cdot\mathbf{c}$ has no meaning.

(b) $(\mathbf{a}\cdot\mathbf{b})\,\mathbf{c}$ is a scalar multiple of a vector, so it does have meaning.

(c) Both $|\mathbf{a}|$ and $\mathbf{b}\cdot\mathbf{c}$ are scalars, so $|\mathbf{a}|\,(\mathbf{b}\cdot\mathbf{c})$ is an ordinary product of real numbers, and has meaning.

(d) Both $\mathbf{a}$ and $\mathbf{b}+\mathbf{c}$ are vectors, so the dot product $\mathbf{a}\cdot(\mathbf{b}+\mathbf{c})$ has meaning.

(e) $\mathbf{a}\cdot\mathbf{b}$ is a scalar, but $\mathbf{c}$ is a vector, and so the two quantities cannot be added and $\mathbf{a}\cdot\mathbf{b}+\mathbf{c}$ has no meaning.

(f) $|\mathbf{a}|$ is a scalar, and the dot product is defined only for vectors, so $|\mathbf{a}|\cdot(\mathbf{b}+\mathbf{c})$ has no meaning.

7. $\mathbf{a}\cdot\mathbf{b}=(2\,\mathbf{i}+\mathbf{j})\cdot(\mathbf{i}-\mathbf{j}+\mathbf{k})=(2)(1)+(1)(-1)+(0)(1)=1$

11. $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are all unit vectors, so the triangle is an equilateral triangle. Thus the angle between $\mathbf{u}$ and $\mathbf{v}$ is 60° and $\mathbf{u}\cdot\mathbf{v}=|\mathbf{u}|\,|\mathbf{v}|\cos 60^\circ=(1)(1)\left(\frac{1}{2}\right)=\frac{1}{2}$. If $\mathbf{w}$ is moved so it has the same initial point as $\mathbf{u}$, we can see that the angle between them is 120° and we have $\mathbf{u}\cdot\mathbf{w}=|\mathbf{u}|\,|\mathbf{w}|\cos 120^\circ=(1)(1)\left(-\frac{1}{2}\right)=-\frac{1}{2}$.

19. $|\mathbf{a}|=\sqrt{4^2+(-3)^2+1^2}=\sqrt{26}$, $|\mathbf{b}|=\sqrt{2^2+0^2+(-1)^2}=\sqrt{5}$, and $\mathbf{a}\cdot\mathbf{b}=(4)(2)+(-3)(0)+(1)(-1)=7$.

Then $\cos\theta=\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|\,|\mathbf{b}|}=\dfrac{7}{\sqrt{26}\cdot\sqrt{5}}=\dfrac{7}{\sqrt{130}}$ and $\theta=\cos^{-1}\left(\dfrac{7}{\sqrt{130}}\right)\approx 52^\circ$.

43. $|\mathbf{a}|=\sqrt{4+1+16}=\sqrt{21}$ so the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\operatorname{comp}_{\mathbf{a}}\mathbf{b}=\dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}=\dfrac{0-1+2}{\sqrt{21}}=\dfrac{1}{\sqrt{21}}$ while the vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is $\operatorname{proj}_{\mathbf{a}}\mathbf{b}=\dfrac{1}{\sqrt{21}}\dfrac{\mathbf{a}}{|\mathbf{a}|}=\dfrac{1}{\sqrt{21}}\cdot\dfrac{2\,\mathbf{i}-\mathbf{j}+4\,\mathbf{k}}{\sqrt{21}}=\frac{1}{21}(2\,\mathbf{i}-\mathbf{j}+4\,\mathbf{k})=\frac{2}{21}\,\mathbf{i}-\frac{1}{21}\,\mathbf{j}+\frac{4}{21}\,\mathbf{k}$.

49.
The displacement vector is $\mathbf{D}=(6-0)\,\mathbf{i}+(12-10)\,\mathbf{j}+(20-8)\,\mathbf{k}=6\,\mathbf{i}+2\,\mathbf{j}+12\,\mathbf{k}$ so, by Equation 12, the work done is $W=\mathbf{F}\cdot\mathbf{D}=(8\,\mathbf{i}-6\,\mathbf{j}+9\,\mathbf{k})\cdot(6\,\mathbf{i}+2\,\mathbf{j}+12\,\mathbf{k})=48-12+108=144$ joules.

53. First note that $\mathbf{n}=\langle a,b\rangle$ is perpendicular to the line, because if $Q_1=(a_1,b_1)$ and $Q_2=(a_2,b_2)$ lie on the line, then $\mathbf{n}\cdot\overrightarrow{Q_1Q_2}=aa_2-aa_1+bb_2-bb_1=0$, since $aa_2+bb_2=-c=aa_1+bb_1$ from the equation of the line.

Let $P_2=(x_2,y_2)$ lie on the line. Then the distance from P_1 to the line is the absolute value of the scalar projection of $\overrightarrow{P_1P_2}$ onto $\mathbf{n}$. $\operatorname{comp}_{\mathbf{n}}\left(\overrightarrow{P_1P_2}\right)=\dfrac{|\mathbf{n}\cdot\langle x_2-x_1,y_2-y_1\rangle|}{|\mathbf{n}|}=\dfrac{|ax_2-ax_1+by_2-by_1|}{\sqrt{a^2+b^2}}=\dfrac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

since $ax_2+by_2=-c$. The required distance is $\dfrac{|(3)(-2)+(-4)(3)+5|}{\sqrt{3^2+(-4)^2}}=\dfrac{13}{5}$.

56.
Consider a cube with sides of unit length, wholly within the first octant and with edges along each of the three coordinate axes. $\mathbf{i}+\mathbf{j}+\mathbf{k}$ and $\mathbf{i}+\mathbf{j}$ are vector representations of a diagonal of the cube and a diagonal of one of its faces. If θ is the angle between these diagonals, then $\cos\theta = \dfrac{(\mathbf{i}+\mathbf{j}+\mathbf{k})\cdot(\mathbf{i}+\mathbf{j})}{|\mathbf{i}+\mathbf{j}+\mathbf{k}|\,|\mathbf{i}+\mathbf{j}|} = \dfrac{1+1}{\sqrt{3}\,\sqrt{2}} = \sqrt{\dfrac{2}{3}} \quad\Rightarrow\quad \theta = \cos^{-1}\sqrt{\frac{2}{3}} \approx 35^\circ$.

4. $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & 7 \\ 2 & -1 & 4 \end{vmatrix} = \begin{vmatrix} 1 & 7 \\ -1 & 4 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 0 & 7 \\ 2 & 4 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 0 & 1 \\ 2 & -1 \end{vmatrix}\mathbf{k}$

$= [4-(-7)]\,\mathbf{i} - (0-14)\,\mathbf{j} + (0-2)\,\mathbf{k} = 11\,\mathbf{i} + 14\,\mathbf{j} - 2\,\mathbf{k}$

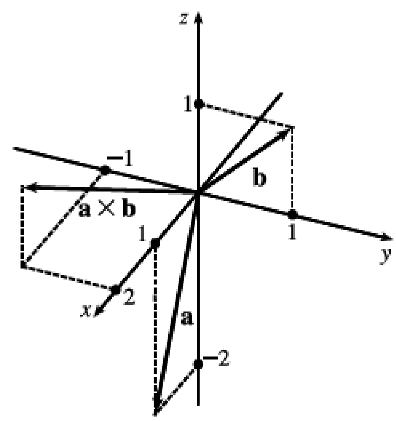
Since $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{a} = (11\,\mathbf{i}+14\,\mathbf{j}-2\,\mathbf{k})\cdot(\mathbf{j}+7\,\mathbf{k}) = 0+14-14 = 0$, $\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{a}$.

Since $(\mathbf{a}\times\mathbf{b})\cdot\mathbf{b} = (11\,\mathbf{i}+14\,\mathbf{j}-2\,\mathbf{k})\cdot(2\,\mathbf{i}-\mathbf{j}+4\,\mathbf{k}) = 22-14-8 = 0$, $\mathbf{a}\times\mathbf{b}$ is orthogonal to $\mathbf{b}$.

8. $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{vmatrix}$

$= \begin{vmatrix} 0 & -2 \\ 1 & 1 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}\mathbf{k}$

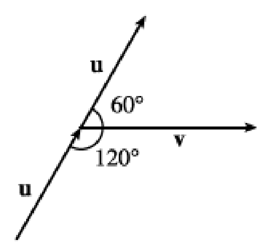
$= 2\,\mathbf{i} - \mathbf{j} + \mathbf{k}$



15. If we sketch $\mathbf{u}$ and $\mathbf{v}$ starting from the same initial point, we see that the angle between them is 60°. Using Theorem 9, we have

$|\mathbf{u}\times\mathbf{v}| = |\mathbf{u}|\,|\mathbf{v}|\sin\theta = (12)(16)\sin 60^\circ = 192\cdot\dfrac{\sqrt{3}}{2} = 96\sqrt{3}$.

By the right-hand rule, $\mathbf{u}\times\mathbf{v}$ is directed into the page.



20. By Theorem 8, the cross product of two vectors is orthogonal to both vectors. So we calculate

$$(\mathbf{j}-\mathbf{k})\times(\mathbf{i}+\mathbf{j}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 0 & -1 \\ 1 & 0 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}\mathbf{k} = \mathbf{i}-\mathbf{j}-\mathbf{k}$$

Thus two unit vectors orthogonal to both given vectors are $\pm\frac{1}{\sqrt{3}}(\mathbf{i}-\mathbf{j}-\mathbf{k})$, that is, $\frac{1}{\sqrt{3}}\,\mathbf{i} - \frac{1}{\sqrt{3}}\,\mathbf{j} - \frac{1}{\sqrt{3}}\,\mathbf{k}$ and $-\frac{1}{\sqrt{3}}\,\mathbf{i} + \frac{1}{\sqrt{3}}\,\mathbf{j} + \frac{1}{\sqrt{3}}\,\mathbf{k}$.

34. $\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c}) = \begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} = 1\begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 0+1+0 = 1.$

So the volume of the parallelepiped determined by $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is 1 cubic unit.

39. The magnitude of the torque is $|\boldsymbol{\tau}| = |\mathbf{r}\times\mathbf{F}| = |\mathbf{r}|\,|\mathbf{F}|\sin\theta = (0.18\text{ m})(60\text{ N})\sin(70+10)^\circ = 10.8\sin 80^\circ \approx 10.6\text{ N}\cdot\text{m}.$

44.

(a) Let $\mathbf{v} = \langle v_1, v_2, v_3\rangle$. Then

$$\langle 1,2,1\rangle \times \mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 1 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} 2 & 1 \\ v_2 & v_3 \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 1 \\ v_1 & v_3 \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 2 \\ v_1 & v_2 \end{vmatrix}\mathbf{k} = (2v_3 - v_2)\,\mathbf{i} - (v_3 - v_1)\,\mathbf{j} + (v_2 - 2v_1)\,\mathbf{k}.$$

If $\langle 1,2,1\rangle \times \mathbf{v} = \langle 3,1,-5\rangle$ then $\langle 2v_3 - v_2, v_1 - v_3, v_2 - 2v_1\rangle = \langle 3,1,-5\rangle \Leftrightarrow 2v_3 - v_2 = 3$ **(1)**, $v_1 - v_3 = 1$ **(2)**, and $v_2 - 2v_1 = -5$ **(3)**. From **(3)** we have $v_2 = 2v_1 - 5$ and from **(2)** we have $v_3 = v_1 - 1$; substitution into **(1)** gives $2(v_1 - 1) - (2v_1 - 5) = 3 \Rightarrow 3 = 3$, so this is a dependent system. If we let $v_1 = a$ then $v_2 = 2a - 5$ and $v_3 = a - 1$, so $\mathbf{v}$ is any vector of the form $\langle a, 2a-5, a-1\rangle$.

(b) If $\langle 1,2,1\rangle \times \mathbf{v} = \langle 3,1,5\rangle$ then $2v_3 - v_2 = 3$ **(1)**, $v_1 - v_3 = 1$ **(2)**, and $v_2 - 2v_1 = 5$ **(3)**. From **(3)** we have $v_2 = 2v_1 + 5$ and from **(2)** we have $v_3 = v_1 - 1$; substitution into **(1)** gives $2(v_1 - 1) - (2v_1 + 5) = 3 \Rightarrow -7 = 3$, so this is an inconsistent system and has no solution.

Alternatively, if we use matrices to solve the system we could show that the determinant is 0 (and hence the system has no solution).