34. The distance from the focus $(3,6)$ to the vertex $(3,2)$ is $6-2=4$. Since the focus is above the vertex, $p=4$.

An equation is $(x-3)^2 = 4p(y-2) \quad \Rightarrow \quad (x-3)^2 = 16(y-2)$.

40. Since the foci are $(0,-1)$ and $(8,-1)$, the ellipse has center $(4,-1)$ with a horizontal axis and $c=4$.

The vertex $(9,-1)$ is 5 units from the center, so $a=5$ and $b=\sqrt{a^2-c^2}=\sqrt{5^2-4^2}=\sqrt{9}$. An equation is

$$\frac{(x-4)^2}{a^2}+\frac{(y+1)^2}{b^2}=1 \quad \Rightarrow \quad \frac{(x-4)^2}{25}+\frac{(y+1)^2}{9}=1.$$

48. The center of a hyperbola with foci $(2,0)$ and $(2,8)$ is $(2,4)$, so $c=4$ and an equation is $\dfrac{(y-4)^2}{a^2}-\dfrac{(x-2)^2}{b^2}=1$.

The asymptote $y=3+\frac{1}{2}x$ has slope $\frac{1}{2}$, so $\dfrac{a}{b}=\dfrac{1}{2} \quad \Rightarrow \quad b=2a$ and $a^2+b^2=c^2 \quad \Rightarrow \quad a^2+(2a)^2=4^2 \quad \Rightarrow$

$5a^2=16 \quad \Rightarrow \quad a^2=\frac{16}{5}$ and so $b^2=16-\frac{16}{5}=\frac{64}{5}$. Thus, an equation is $\dfrac{(y-4)^2}{16/5}-\dfrac{(x-2)^2}{64/5}=1$.

2. For this line, we have $\mathbf{r}_0 = 6\,\mathbf{i}-5\,\mathbf{j}+2\,\mathbf{k}$ and $\mathbf{v}=\mathbf{i}+3\,\mathbf{j}-\frac{2}{3}\,\mathbf{k}$, so a vector equation is

$\mathbf{r}=\mathbf{r}_0+t\,\mathbf{v}=(6\,\mathbf{i}-5\,\mathbf{j}+2\,\mathbf{k})+t\big(\mathbf{i}+3\,\mathbf{j}-\frac{2}{3}\,\mathbf{k}\big)=(6+t)\,\mathbf{i}+(-5+3t)\,\mathbf{j}+\big(2-\frac{2}{3}t\big)\,\mathbf{k}$ and parametric equations are

$x=6+t$, $y=-5+3t$, $z=2-\frac{2}{3}t$.

5. A line perpendicular to the given plane has the same direction as a normal vector to the plane, such as

$\mathbf{n}=\langle 1,3,1\rangle$. So $\mathbf{r}_0=\mathbf{i}+6\,\mathbf{k}$, and we can take $\mathbf{v}=\mathbf{i}+3\,\mathbf{j}+\mathbf{k}$. Then a vector equation is

$\mathbf{r}=(\mathbf{i}+6\,\mathbf{k})+t(\mathbf{i}+3\,\mathbf{j}+\mathbf{k})=(1+t)\,\mathbf{i}+3t\,\mathbf{j}+(6+t)\,\mathbf{k}$, and parametric equations are $x=1+t$, $y=3t$, $z=6+t$.

9. $\mathbf{v}=\langle 3-(-8),-2-1,4-4\rangle=\langle 11,-3,0\rangle$, and letting $P_0=(-8,1,4)$, parametric equations are $x=-8+11t$,

$y=1-3t$, $z=4+0t=4$, while symmetric equations are $\dfrac{x+8}{11}=\dfrac{y-1}{-3}$, $z=4$. Notice here that the direction number

$c=0$, so rather than writing $\dfrac{z-4}{0}$ in the symmetric equation we must write the equation $z=4$ separately.

12. Setting $z=0$ we see that $(1,0,0)$ satisfies the equations of both planes, so they do in fact have a line of intersection.

The line is perpendicular to the normal vectors of both planes, so a direction vector for the line is

$\mathbf{v}=\mathbf{n}_1\times\mathbf{n}_2=\langle 1,2,3\rangle\times\langle 1,-1,1\rangle=\langle 5,2,-3\rangle$. Taking the point $(1,0,0)$ as P_0, parametric equations are $x=1+5t$,

$y=2t$, $z=-3t$, and symmetric equations are $\dfrac{x-1}{5}=\dfrac{y}{2}=\dfrac{z}{-3}$.

14.

Direction vectors of the lines are $\mathbf{v}_1=\langle 3,-3,1\rangle$ and $\mathbf{v}_2=\langle 1,-4,-12\rangle$. Since $\mathbf{v}_1\cdot\mathbf{v}_2=3+12-12\neq 0$, the vectors and thus the lines are not perpendicular.

18. From Equation 4, the line segment from $\mathbf{r}_0 = 10\,\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}$ to $\mathbf{r}_1 = 5\,\mathbf{i} + 6\,\mathbf{j} - 3\,\mathbf{k}$ is

$$\begin{aligned}\mathbf{r}(t) &= (1-t)\,\mathbf{r}_0 + t\,\mathbf{r}_1 = (1-t)(10\,\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}) + t(5\,\mathbf{i} + 6\,\mathbf{j} - 3\,\mathbf{k}) \\ &= (10\,\mathbf{i} + 3\,\mathbf{j} + \mathbf{k}) + t(-5\,\mathbf{i} + 3\,\mathbf{j} - 4\,\mathbf{k}), \quad 0 \le t \le 1.\end{aligned}$$

The corresponding parametric equations are $x = 10 - 5t$, $y = 3 + 3t$, $z = 1 - 4t$, $0 \le t \le 1$.

21.
Since the direction vectors $\langle 1, -2, -3\rangle$ and $\langle 1, 3, -7\rangle$ aren't scalar multiples of each other, the lines aren't parallel. Parametric equations of the lines are L_1: $x = 2 + t$, $y = 3 - 2t$, $z = 1 - 3t$ and L_2: $x = 3 + s$, $y = -4 + 3s$, $z = 2 - 7s$. Thus, for the lines to intersect, the three equations $2 + t = 3 + s$, $3 - 2t = -4 + 3s$, and $1 - 3t = 2 - 7s$ must be satisfied simultaneously. Solving the first two equations gives $t = 2$, $s = 1$ and checking, we see that these values do satisfy the third equation, so the lines intersect when $t = 2$ and $s = 1$, that is, at the point $(4, -1, -5)$.

23. Since the plane is perpendicular to the vector $\langle 1, -2, 5\rangle$, we can take $\langle 1, -2, 5\rangle$ as a normal vector to the plane. $(0, 0, 0)$ is a point on the plane, so setting $a = 1$, $b = -2$, $c = 5$ and $x_0 = 0$, $y_0 = 0$, $z_0 = 0$ in Equation 7 gives $1(x - 0) + (-2)(y - 0) + 5(z - 0) = 0$ or $x - 2y + 5z = 0$ as an equation of the plane.

28. Since the two planes are parallel, they will have the same normal vectors. A normal vector for the plane $z = x + y$ or $x + y - z = 0$ is $\mathbf{n} = \langle 1, 1, -1\rangle$, and an equation of the desired plane is $1(x - 2) + 1(y - 4) - 1(z - 6) = 0$ or $x + y - z = 0$ (the same plane!).

34.
If we first find two nonparallel vectors in the plane, their cross product will be a normal vector to the plane. Since the given line lies in the plane, its direction vector $\mathbf{a} = \langle 3, 1, -1\rangle$ is one vector in the plane. We can verify that the given point $(1, 2, 3)$ does not lie on this line, so to find another nonparallel vector $\mathbf{b}$ which lies in the plane, we can pick any point on the line and find a vector connecting the points. If we put $t = 0$, we see that $(0, 1, 2)$ is on the line, so $\mathbf{b} = \langle 1 - 0, 2 - 1, 3 - 2\rangle = \langle 1, 1, 1\rangle$ and $\mathbf{n} = \mathbf{a} \times \mathbf{b} = \langle 1 + 1, -1 - 3, 3 - 1\rangle = \langle 2, -4, 2\rangle$. Thus, an equation of the plane is $2(x - 1) - 4(y - 2) + 2(z - 3) = 0$ or $2x - 4y + 2z = 0$. (Equivalently, we can write $x - 2y + z = 0$.)

39.
If a plane is perpendicular to two other planes, its normal vector is perpendicular to the normal vectors of the other two planes. Thus $\langle 2, 1, -2\rangle \times \langle 1, 0, 3\rangle = \langle 3 - 0, -2 - 6, 0 - 1\rangle = \langle 3, -8, -1\rangle$ is a normal vector to the desired plane. The point $(1, 5, 1)$ lies on the plane, so an equation is $3(x - 1) - 8(y - 5) - (z - 1) = 0$ or $3x - 8y - z = -38$.

47. Parametric equations for the line are $x = t$, $y = 1 + t$, $z = \frac{1}{2}t$ and substituting into the equation of the plane gives $4(t) - (1 + t) + 3\left(\frac{1}{2}t\right) = 8 \;\Rightarrow\; \frac{9}{2}t = 9 \;\Rightarrow\; t = 2$. Thus $x = 2$, $y = 1 + 2 = 3$, $z = \frac{1}{2}(2) = 1$ and the point of intersection is $(2, 3, 1)$.

49.

Setting $x = 0$, we see that $(0, 1, 0)$ satisfies the equations of both planes, so that they do in fact have a line of intersection.

$\mathbf{v} = \mathbf{n}_1 \times \mathbf{n}_2 = \langle 1, 1, 1 \rangle \times \langle 1, 0, 1 \rangle = \langle 1, 0, -1 \rangle$ is the direction of this line. Therefore, direction numbers of the intersecting line are 1, 0, -1.