2. The component functions $\frac{t-2}{t+2}$, $\sin t$, and $\ln(9-t^2)$ are all defined when $t \neq -2$ and $9 - t^2 > 0 \quad \Rightarrow \quad -3 < t < 3$, so the domain of $\mathbf{r}$ is $(-3,-2) \cup (-2,3)$.

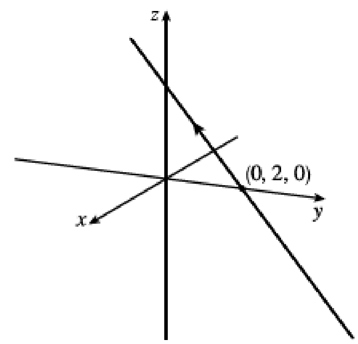
4. $\lim_{t \to 1} \frac{t^2 - t}{t-1} = \lim_{t \to 1} \frac{t\,(t-1)}{t-1} = \lim_{t \to 1} t = 1$, $\lim_{t \to 1} \sqrt{t+8} = 3$, $\lim_{t \to 1} \frac{\sin \pi t}{\ln t} = \lim_{t \to 1} \frac{\pi \cos \pi t}{1/t} = -\pi$ [by l'Hospital's Rule]. Thus the given limit equals $\mathbf{i} + 3\,\mathbf{j} - \pi\,\mathbf{k}$.

6. $\lim_{t \to \infty} te^{-t} = \lim_{t \to \infty} \frac{t}{e^t} = \lim_{t \to \infty} \frac{1}{e^t} = 0$ [by l'Hospital's Rule], $\lim_{t \to \infty} \frac{t^3+t}{2t^3-1} = \lim_{t \to \infty} \frac{1+(1/t^2)}{2-(1/t^3)} = \frac{1+0}{2-0} = \frac{1}{2}$,

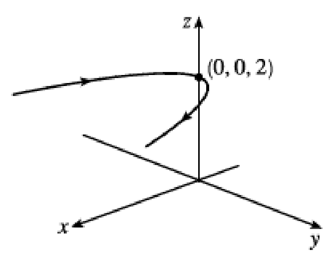
and $\lim_{t \to \infty} t \sin \frac{1}{t} = \lim_{t \to \infty} \frac{\sin(1/t)}{1/t} = \lim_{t \to \infty} \frac{\cos(1/t)(-1/t^2)}{-1/t^2} = \lim_{t \to \infty} \cos \frac{1}{t} = \cos 0 = 1$ [again by l'Hospital's Rule].

Thus $\lim_{t \to \infty} \left\langle te^{-t}, \frac{t^3+t}{2t^3-1}, t \sin \frac{1}{t} \right\rangle = \left\langle 0, \frac{1}{2}, 1 \right\rangle$.

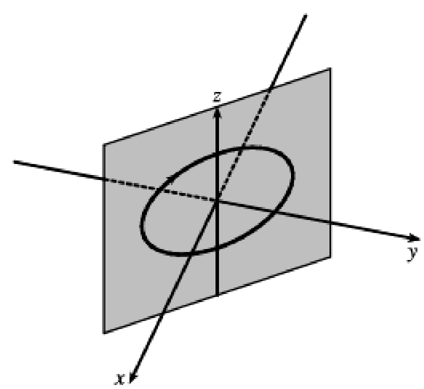
9. The corresponding parametric equations are $x = t,\ y = 2 - t,\ z = 2t$, which are parametric equations of a line through the point $(0, 2, 0)$ and with direction vector $\langle 1, -1, 2 \rangle$.



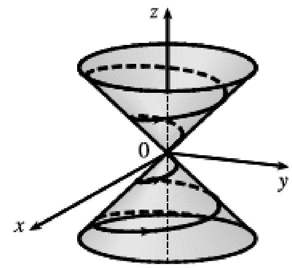
12. The parametric equations are $x = t^2$, $y = t$, $z = 2$, so we have $x = y^2$ with $z = 2$. Thus the curve is a parabola in the plane $z = 2$ with vertex $(0, 0, 2)$.



14. If $x = \cos t,\ y = -\cos t,\ z = \sin t$, then $x^2 + z^2 = 1$ and $y^2 + z^2 = 1$, so the curve is contained in the intersection of circular cylinders along the x- and y-axes. Furthermore, $y = -x$, so the curve is an ellipse in the plane $y = -x$, centered at the origin.



27. If $x = t\cos t,\ y = t\sin t,\ z = t$, then $x^2 + y^2 = t^2\cos^2 t + t^2\sin^2 t = t^2 = z^2$, so the curve lies on the cone $z^2 = x^2 + y^2$. Since $z = t$, the curve is a spiral on this cone.



29. Parametric equations for the curve are $x = t,\ y = 0,\ z = 2t - t^2$. Substituting into the equation of the paraboloid gives $2t - t^2 = t^2 \Rightarrow 2t = 2t^2 \Rightarrow t = 0, 1$. Since $\mathbf{r}(0) = \mathbf{0}$ and $\mathbf{r}(1) = \mathbf{i} + \mathbf{k}$, the points of intersection are $(0, 0, 0)$ and $(1, 0, 1)$.

40. The projection of the curve C of intersection onto the xy-plane is the circle $x^2 + y^2 = 4$, $z = 0$. Then we can write $x = 2\cos t,\ y = 2\sin t,\ 0 \le t \le 2\pi$. Since C also lies on the surface $z = xy$, we have $z = xy = (2\cos t)(2\sin t) = 4\cos t\sin t$, or $2\sin(2t)$. Then parametric equations for C are $x = 2\cos t,\ y = 2\sin t,\ z = 2\sin(2t),\ 0 \le t \le 2\pi$, and the corresponding vector function is $\mathbf{r}(t) = 2\cos t\,\mathbf{i} + 2\sin t\,\mathbf{j} + 2\sin(2t)\,\mathbf{k},\ 0 \le t \le 2\pi$.

42. The projection of the curve C of intersection onto the xy-plane is the parabola $y = x^2$, $z = 0$. Then we can choose the parameter $x = t \Rightarrow y = t^2$. Since C also lies on the surface $z = 4x^2 + y^2$, we have $z = 4x^2 + y^2 = 4t^2 + (t^2)^2$. Then parametric equations for C are $x = t,\ y = t^2,\ z = 4t^2 + t^4$, and the corresponding vector function is $\mathbf{r}(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + (4t^2 + t^4)\,\mathbf{k}$.

48.

The particles collide provided $\mathbf{r}_1(t) = \mathbf{r}_2(t) \Leftrightarrow \langle t, t^2, t^3\rangle = \langle 1 + 2t, 1 + 6t, 1 + 14t\rangle$. Equating components gives $t = 1 + 2t$, $t^2 = 1 + 6t$, and $t^3 = 1 + 14t$. The first equation gives $t = -1$, but this does not satisfy the other equations, so the particles do not collide. For the paths to intersect, we need to find a value for t and a value for s where $\mathbf{r}_1(t) = \mathbf{r}_2(s) \Leftrightarrow \langle t, t^2, t^3\rangle = \langle 1 + 2s, 1 + 6s, 1 + 14s\rangle$. Equating components, $t = 1 + 2s$, $t^2 = 1 + 6s$, and $t^3 = 1 + 14s$. Substituting the first equation into the second gives $(1 + 2s)^2 = 1 + 6s \Rightarrow 4s^2 - 2s = 0 \Rightarrow 2s(2s - 1) = 0 \Rightarrow s = 0$ or $s = \frac{1}{2}$. From the first equation, $s = 0 \Rightarrow t = 1$ and $s = \frac{1}{2} \Rightarrow t = 2$. Checking, we see that both pairs of values satisfy the third equation. Thus the paths intersect twice, at the point $(1, 1, 1)$ when $s = 0$ and $t = 1$, and at $(2, 4, 8)$ when $s = \frac{1}{2}$ and $t = 2$.