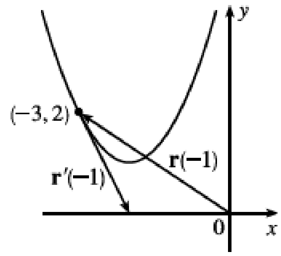
3. Since $(x+2)^2 = t^2 = y-1 \quad \Rightarrow$ $y = (x+2)^2 + 1$, the curve is a parabola.

(a), (c)

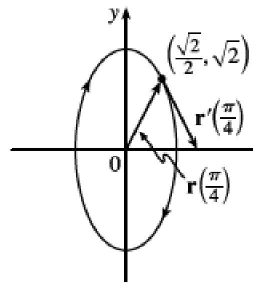


(b) $\mathbf{r}'(t) = \langle 1, 2t \rangle$,

$\mathbf{r}'(-1) = \langle 1, -2 \rangle$

5. $x = \sin t, \quad y = 2\cos t$ so $x^2 + (y/2)^2 = 1$ and the curve is an ellipse.

(a), (c)

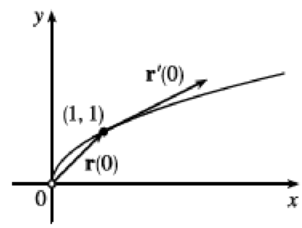


(b) $\mathbf{r}'(t) = \cos t\,\mathbf{i} - 2\sin t\,\mathbf{j}$,

$$\mathbf{r}'\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}\,\mathbf{i} - \sqrt{2}\,\mathbf{j}$$

7. Since $x = e^{2t} = (e^t)^2 = y^2$, the curve is part of a parabola. Note that here $x > 0$, $y > 0$.

(a), (c)



(b) $\mathbf{r}'(t) = 2e^{2t}\,\mathbf{i} + e^t\,\mathbf{j}$,

$\mathbf{r}'(0) = 2\,\mathbf{i} + \mathbf{j}$

9. $$\mathbf{r}'(t) = \left\langle \frac{d}{dt}\left[t\sin t\right], \frac{d}{dt}\left[t^2\right], \frac{d}{dt}\left[t\cos 2t\right] \right\rangle = \langle t\cos t + \sin t, 2t, t(-\sin 2t)\cdot 2 + \cos 2t \rangle$$

$$= \langle t\cos t + \sin t, 2t, \cos 2t - 2t\sin 2t \rangle$$

21. $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle 1, 2t, 3t^2 \rangle$. Then $\mathbf{r}'(1) = \langle 1, 2, 3 \rangle$ and $|\mathbf{r}'(1)| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$, so

$\mathbf{T}(1) = \dfrac{\mathbf{r}'(1)}{|\mathbf{r}'(1)|} = \frac{1}{\sqrt{14}}\langle 1, 2, 3 \rangle = \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$. $\quad \mathbf{r}''(t) = \langle 0, 2, 6t \rangle$, so

$$\mathbf{r}'(t) \times \mathbf{r}''(t) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix} = \begin{vmatrix} 2t & 3t^2 \\ 2 & 6t \end{vmatrix}\mathbf{i} - \begin{vmatrix} 1 & 3t^2 \\ 0 & 6t \end{vmatrix}\mathbf{j} + \begin{vmatrix} 1 & 2t \\ 0 & 2 \end{vmatrix}\mathbf{k}$$

$$= (12t^2 - 6t^2)\,\mathbf{i} - (6t - 0)\,\mathbf{j} + (2 - 0)\,\mathbf{k} = \langle 6t^2, -6t, 2 \rangle$$

27. First we parametrize the curve C of intersection. The projection of C onto the xy-plane is contained in the circle $x^2 + y^2 = 25$, $z = 0$, so we can write $x = 5\cos t$, $y = 5\sin t$. C also lies on the cylinder $y^2 + z^2 = 20$, and $z \geq 0$ near the point $(3, 4, 2)$, so we can write $z = \sqrt{20 - y^2} = \sqrt{20 - 25\sin^2 t}$. A vector equation then for C is

$$\mathbf{r}(t) = \left\langle 5\cos t, 5\sin t, \sqrt{20 - 25\sin^2 t} \right\rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \left\langle -5\sin t, 5\cos t, \tfrac{1}{2}(20 - 25\sin^2 t)^{-1/2}(-50\sin t\cos t) \right\rangle.$$

The point $(3, 4, 2)$ corresponds to $t = \cos^{-1}\left(\frac{3}{5}\right)$, so the tangent vector there is

$$\mathbf{r}'\left(\cos^{-1}\left(\tfrac{3}{5}\right)\right) = \left\langle -5\left(\tfrac{4}{5}\right), 5\left(\tfrac{3}{5}\right), \tfrac{1}{2}\left(20 - 25\left(\tfrac{4}{5}\right)^2\right)^{-1/2}\left(-50\left(\tfrac{4}{5}\right)\left(\tfrac{3}{5}\right)\right) \right\rangle = \langle -4, 3, -6 \rangle.$$

The tangent line is parallel to this vector and passes through $(3, 4, 2)$, so a vector equation for the line is $\mathbf{r}(t) = (3 - 4t)\mathbf{i} + (4 + 3t)\mathbf{j} + (2 - 6t)\mathbf{k}$.

28.
$\mathbf{r}(t) = \langle 2\cos t, 2\sin t, e^t \rangle \quad \Rightarrow \quad \mathbf{r}'(t) = \langle -2\sin t, 2\cos t, e^t \rangle$. The tangent line to the curve is parallel to the plane when the curve's tangent vector is orthogonal to the plane's normal vector. Thus we require $\langle -2\sin t, 2\cos t, e^t \rangle \cdot \langle \sqrt{3}, 1, 0 \rangle = 0 \quad \Rightarrow$ $-2\sqrt{3}\sin t + 2\cos t + 0 = 0 \quad \Rightarrow \quad \tan t = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad t = \frac{\pi}{6}$ [since $0 \leq t \leq \pi$].

$\mathbf{r}\left(\frac{\pi}{6}\right) = \left\langle \sqrt{3}, 1, e^{\pi/6} \right\rangle$, so the point is $(\sqrt{3}, 1, e^{\pi/6})$.

33.
The angle of intersection of the two curves is the angle between the two tangent vectors to the curves at the point of intersection. Since $\mathbf{r}_1'(t) = \langle 1, 2t, 3t^2 \rangle$ and $t = 0$ at $(0, 0, 0)$, $\mathbf{r}_1'(0) = \langle 1, 0, 0 \rangle$ is a tangent vector to $\mathbf{r}_1$ at $(0, 0, 0)$. Similarly, $\mathbf{r}_2'(t) = \langle \cos t, 2\cos 2t, 1 \rangle$ and since $\mathbf{r}_2(0) = \langle 0, 0, 0 \rangle$, $\mathbf{r}_2'(0) = \langle 1, 2, 1 \rangle$ is a tangent vector to $\mathbf{r}_2$ at $(0, 0, 0)$. If θ is the angle between these two tangent vectors, then $\cos\theta = \frac{1}{\sqrt{1}\sqrt{6}} \langle 1, 0, 0 \rangle \cdot \langle 1, 2, 1 \rangle = \frac{1}{\sqrt{6}}$ and $\theta = \cos^{-1}\left(\frac{1}{\sqrt{6}}\right) \approx 66°$.

37. $\int_0^{\pi/2} (3\sin^2 t\, \cos t\, \mathbf{i} + 3\sin t\, \cos^2 t\, \mathbf{j} + 2\sin t\, \cos t\, \mathbf{k})\, dt$

$$= \left(\int_0^{\pi/2} 3\sin^2 t\, \cos t\, dt\right)\mathbf{i} + \left(\int_0^{\pi/2} 3\sin t\, \cos^2 t\, dt\right)\mathbf{j} + \left(\int_0^{\pi/2} 2\sin t\, \cos t\, dt\right)\mathbf{k}$$

$$= \left[\sin^3 t\right]_0^{\pi/2}\mathbf{i} + \left[-\cos^3 t\right]_0^{\pi/2}\mathbf{j} + \left[\sin^2 t\right]_0^{\pi/2}\mathbf{k} = (1 - 0)\,\mathbf{i} + (0 + 1)\,\mathbf{j} + (1 - 0)\,\mathbf{k} = \mathbf{i} + \mathbf{j} + \mathbf{k}$$

41. $\mathbf{r}'(t) = 2t\,\mathbf{i} + 3t^2\,\mathbf{j} + \sqrt{t}\,\mathbf{k} \quad \Rightarrow \quad \mathbf{r}(t) = t^2\,\mathbf{i} + t^3\,\mathbf{j} + \frac{2}{3}t^{3/2}\,\mathbf{k} + \mathbf{C}$, where $\mathbf{C}$ is a constant vector.

But $\mathbf{i} + \mathbf{j} = \mathbf{r}(1) = \mathbf{i} + \mathbf{j} + \frac{2}{3}\mathbf{k} + \mathbf{C}$. Thus $\mathbf{C} = -\frac{2}{3}\mathbf{k}$ and $\mathbf{r}(t) = t^2\,\mathbf{i} + t^3\,\mathbf{j} + \left(\frac{2}{3}t^{3/2} - \frac{2}{3}\right)\mathbf{k}$.

51. $\dfrac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}'(t) \times \mathbf{r}'(t) + \mathbf{r}(t) \times \mathbf{r}''(t)$ by Formula 5 of Theorem 3. But $\mathbf{r}'(t) \times \mathbf{r}'(t) = \mathbf{0}$ (by Example 2 in Section 12.4). Thus, $\dfrac{d}{dt}\left[\mathbf{r}(t) \times \mathbf{r}'(t)\right] = \mathbf{r}(t) \times \mathbf{r}''(t)$.

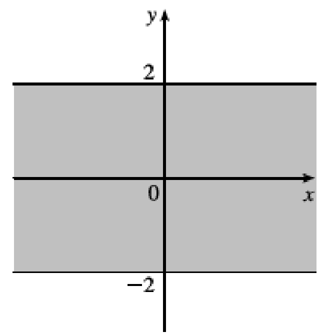
9. (a) $g(2,-1) = \cos(2 + 2(-1)) = \cos(0) = 1$

(b) $x + 2y$ is defined for all choices of values for x and y and the cosine function is defined for all input values, so the domain of g is $\mathbb{R}^2$.

(c) The range of the cosine function is $[-1, 1]$ and $x + 2y$ generates all possible input values for the cosine function, so the range of $\cos(x + 2y)$ is $[-1, 1]$.

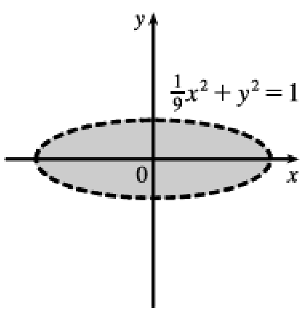
10. (a) $F(3,1) = 1 + \sqrt{4 - 1^2} = 1 + \sqrt{3}$

(b) $\sqrt{4 - y^2}$ is defined only when $4 - y^2 \geq 0$, or $y^2 \leq 4 \quad \Leftrightarrow$ $-2 \leq y \leq 2$. So the domain of F is $\{(x,y) \mid -2 \leq y \leq 2\}$.



(c) We know $0 \leq \sqrt{4 - y^2} \leq 2$ so $1 \leq 1 + \sqrt{4 - y^2} \leq 3$. Thus the range of F is $[1, 3]$.

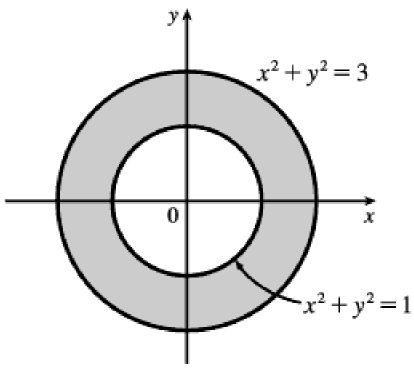
15. $\ln(9 - x^2 - 9y^2)$ is defined only when $9 - x^2 - 9y^2 > 0$, or $\frac{1}{9}x^2 + y^2 < 1$. So the domain of f is $\left\{(x,y) \mid \frac{1}{9}x^2 + y^2 < 1\right\}$, the interior of an ellipse.



20. $\arcsin(x^2 + y^2 - 2)$ is defined only when

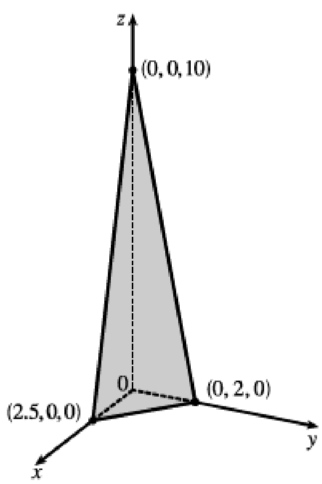
$-1 \le x^2 + y^2 - 2 \le 1 \quad \Leftrightarrow \quad 1 \le x^2 + y^2 \le 3$. Thus

the domain of f is $\{(x, y) \mid 1 \le x^2 + y^2 \le 3\}$.



25. $z = 10 - 4x - 5y$ or $4x + 5y + z = 10$, a plane with

intercepts 2.5, 2, and 10.



30. $z = \sqrt{4x^2 + y^2}$ so $4x^2 + y^2 = z^2$ and $z \ge 0$, the top

half of an elliptic cone.