11. (a) $f(1,1,1) = \sqrt{1} + \sqrt{1} + \sqrt{1} + \ln(4 - 1^2 - 1^2 - 1^2) = 3 + \ln 1 = 3$

(b) $\sqrt{x}$, $\sqrt{y}$, $\sqrt{z}$ are defined only when $x \geq 0$, $y \geq 0$, $z \geq 0$, and $\ln(4 - x^2 - y^2 - z^2)$ is defined when

$4 - x^2 - y^2 - z^2 > 0 \quad \Leftrightarrow \quad x^2 + y^2 + z^2 < 4$, thus the domain is

$\{(x,y,z) \mid x^2 + y^2 + z^2 < 4,\ x \geq 0,\ y \geq 0,\ z \geq 0\}$, the portion of the interior of a sphere of radius 2, centered at the origin, that is in the first octant.

32.
All six graphs have different traces in the planes $x = 0$ and $y = 0$, so we investigate these for each function.

(a) $f(x,y) = |x| + |y|$. The trace in $x = 0$ is $z = |y|$, and in $y = 0$ is $z = |x|$, so it must be graph VI.

(b) $f(x,y) = |xy|$. The trace in $x = 0$ is $z = 0$, and in $y = 0$ is $z = 0$, so it must be graph V.

(c) $f(x,y) = \dfrac{1}{1 + x^2 + y^2}$. The trace in $x = 0$ is $z = \dfrac{1}{1 + y^2}$, and in $y = 0$ is $z = \dfrac{1}{1 + x^2}$. In addition, we can see that f is close to 0 for large values of x and y, so this is graph I.

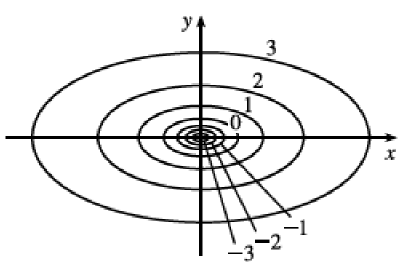
(d) $f(x,y) = (x^2 - y^2)^2$. The trace in $x = 0$ is $z = y^4$, and in $y = 0$ is $z = x^4$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x^2 - y^2)^2 \quad \Rightarrow \quad y = \pm x$, so it must be graph IV.

(e) $f(x,y) = (x - y)^2$. The trace in $x = 0$ is $z = y^2$, and in $y = 0$ is $z = x^2$. Both graph II and graph IV seem plausible; notice the trace in $z = 0$ is $0 = (x - y)^2 \quad \Rightarrow \quad y = x$, so it must be graph II.

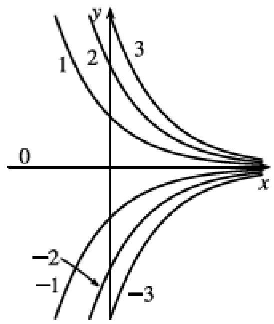
(f) $f(x,y) = \sin(|x| + |y|)$. The trace in $x = 0$ is $z = \sin|y|$, and in $y = 0$ is $z = \sin|x|$. In addition, notice that the oscillating nature of the graph is characteristic of trigonometric functions. So this is graph III.

46. The level curves are $\ln(x^2 + 4y^2) = k$ or $x^2 + 4y^2 = e^k$,

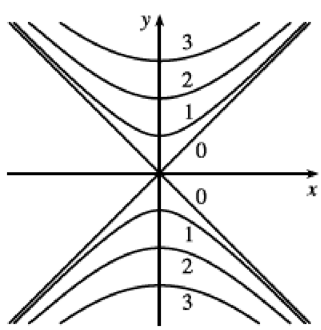
a family of ellipses.



47. The level curves are $ye^x = k$ or $y = ke^{-x}$, a family of exponential curves.



49. The level curves are $\sqrt{y^2 - x^2} = k$ or $y^2 - x^2 = k^2$, $k \geq 0$. When $k = 0$ the level curve is the pair of lines $y = \pm x$. For $k > 0$, the level curves are hyperbolas with axis the y-axis.



66. $k = x^2 + 3y^2 + 5z^2$ is a family of ellipsoids for $k > 0$ and the origin for $k = 0$.

67. Equations for the level surfaces are $k = y^2 + z^2$. For $k > 0$, we have a family of circular cylinders with axis the x-axis and radius $\sqrt{k}$. When $k = 0$ the level surface is the x-axis. (There are no level surfaces for $k < 0$.)

8. $\dfrac{1+y^2}{x^2+xy}$ is a rational function and hence continuous on its domain, which includes $(1, 0)$. $\ln t$ is a continuous function for $t > 0$, so the composition $f(x, y) = \ln\left(\dfrac{1+y^2}{x^2+xy}\right)$ is continuous wherever $\dfrac{1+y^2}{x^2+xy} > 0$. In particular, f is continuous at $(1, 0)$ and so $\lim\limits_{(x,y)\to(1,0)} f(x, y) = f(1, 0) = \ln\left(\dfrac{1+0^2}{1^2+1\cdot 0}\right) = \ln\dfrac{1}{1} = 0$.

9. $f(x, y) = (x^4 - 4y^2)/(x^2 + 2y^2)$. First approach $(0, 0)$ along the x-axis. Then $f(x, 0) = x^4/x^2 = x^2$ for $x \neq 0$, so $f(x, y) \to 0$. Now approach $(0, 0)$ along the y-axis. For $y \neq 0$, $f(0, y) = -4y^2/2y^2 = -2$, so $f(x, y) \to -2$. Since f has two different limits along two different lines, the limit does not exist.

16. We can use the Squeeze Theorem to show that $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2} = 0$:

$0 \le \dfrac{x^2 \sin^2 y}{x^2 + 2y^2} \le \sin^2 y$ since $\dfrac{x^2}{x^2 + 2y^2} \le 1$, and $\sin^2 y \to 0$ as $(x, y) \to (0, 0)$, so $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2} = 0$.

17.
$$\begin{aligned}\lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} &= \lim_{(x,y)\to(0,0)} \frac{x^2 + y^2}{\sqrt{x^2 + y^2 + 1} - 1} \cdot \frac{\sqrt{x^2 + y^2 + 1} + 1}{\sqrt{x^2 + y^2 + 1} + 1} \\ &= \lim_{(x,y)\to(0,0)} \frac{(x^2 + y^2)\left(\sqrt{x^2 + y^2 + 1} + 1\right)}{x^2 + y^2} = \lim_{(x,y)\to(0,0)} \left(\sqrt{x^2 + y^2 + 1} + 1\right) = 2\end{aligned}$$

21. $f(x, y, z) = \dfrac{xy + yz^2 + xz^2}{x^2 + y^2 + z^4}$. Then $f(x, 0, 0) = 0/x^2 = 0$ for $x \neq 0$, so as $(x, y, z) \to (0, 0, 0)$ along the x-axis, $f(x, y, z) \to 0$. But $f(x, x, 0) = x^2/(2x^2) = \frac{1}{2}$ for $x \neq 0$, so as $(x, y, z) \to (0, 0, 0)$ along the line $y = x$, $z = 0$, $f(x, y, z) \to \frac{1}{2}$. Thus the limit doesn't exist.

26. $h(x, y) = g(f(x, y)) = \dfrac{1 - xy}{1 + x^2y^2} + \ln\left(\dfrac{1 - xy}{1 + x^2y^2}\right)$. f is a rational function, so it is continuous on its domain. Because $1 + x^2y^2 > 0$, the domain of f is $\mathbb{R}^2$, so f is continuous everywhere. g is continuous on its domain $\{t \mid t > 0\}$. Thus h is continuous on its domain $\left\{(x, y) \;\middle|\; \dfrac{1 - xy}{1 + x^2y^2} > 0\right\} = \{(x, y) \mid xy < 1\}$ which consists of all points between (but not on) the two branches of the hyperbola $y = 1/x$.

38.

$$f(x, y) = \begin{cases} \dfrac{xy}{x^2 + xy + y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$$

The first piece of f is a rational function defined everywhere except at the origin, so f is continuous on $\mathbb{R}^2$ except possibly at the origin. $f(x, 0) = 0/x^2 = 0$ for $x \neq 0$, so $f(x, y) \to 0$ as $(x, y) \to (0, 0)$ along the x-axis. But $f(x, x) = x^2/(3x^2) = \frac{1}{3}$ for $x \neq 0$, so $f(x, y) \to \frac{1}{3}$ as $(x, y) \to (0, 0)$ along the line $y = x$. Thus $\displaystyle\lim_{(x,y)\to(0,0)} f(x, y)$ doesn't exist, so f is not continuous at $(0, 0)$ and the largest set on which f is continuous is $\{(x, y) \mid (x, y) \neq (0, 0)\}$.

29.

$F(x,y)=\int_y^x \cos(e^t)\,dt \quad\Rightarrow\quad F_x(x,y)=\frac{\partial}{\partial x}\int_y^x \cos(e^t)\;dt=\cos(e^x)$ by the Fundamental Theorem of Calculus, Part 1;

$F_y(x,y)=\frac{\partial}{\partial y}\int_y^x \cos(e^t)\;dt=\frac{\partial}{\partial y}\left[-\int_x^y \cos(e^t)\;dt\right]=-\frac{\partial}{\partial y}\int_x^y \cos(e^t)\;dt=-\cos(e^y)$.

32. $f(x,y,z)=x\sin(y-z) \quad\Rightarrow\quad f_x(x,y,z)=\sin(y-z),\quad f_y(x,y,z)=x\cos(y-z),$

$f_z(x,y,z)=x\cos(y-z)(-1)=-x\cos(y-z)$

33. $w=\ln(x+2y+3z) \quad\Rightarrow\quad \frac{\partial w}{\partial x}=\frac{1}{x+2y+3z},\ \frac{\partial w}{\partial y}=\frac{2}{x+2y+3z},\ \frac{\partial w}{\partial z}=\frac{3}{x+2y+3z}$

34. $w=ze^{xyz} \quad\Rightarrow$

$$\frac{\partial w}{\partial x}=ze^{xyz}\cdot yz=yz^2e^{xyz},\quad \frac{\partial w}{\partial y}=ze^{xyz}\cdot xz=xz^2e^{xyz},\quad \frac{\partial w}{\partial z}=ze^{xyz}\cdot xy+e^{xyz}\cdot 1=(xyz+1)e^{xyz}$$

35.

$$u=xy\sin^{-1}(yz) \quad\Rightarrow\quad \frac{\partial u}{\partial x}=y\sin^{-1}(yz),\quad \frac{\partial u}{\partial y}=xy\cdot\frac{1}{\sqrt{1-(yz)^2}}\,(z)+\sin^{-1}(yz)\cdot x=\frac{xyz}{\sqrt{1-y^2z^2}}+x\sin^{-1}(yz),$$

$$\frac{\partial u}{\partial z}=xy\cdot\frac{1}{\sqrt{1-(yz)^2}}\,(y)=\frac{xy^2}{\sqrt{1-y^2z^2}}$$

55. $w=\sqrt{u^2+v^2} \quad\Rightarrow\quad w_u=\frac{1}{2}(u^2+v^2)^{-1/2}\cdot 2u=\frac{u}{\sqrt{u^2+v^2}},\ w_v=\frac{1}{2}(u^2+v^2)^{-1/2}\cdot 2v=\frac{v}{\sqrt{u^2+v^2}}$. Then

$$w_{uu}=\frac{1\cdot\sqrt{u^2+v^2}-u\cdot\frac{1}{2}(u^2+v^2)^{-1/2}(2u)}{\left(\sqrt{u^2+v^2}\right)^2}=\frac{\sqrt{u^2+v^2}-u^2/\sqrt{u^2+v^2}}{u^2+v^2}=\frac{u^2+v^2-u^2}{(u^2+v^2)^{3/2}}=\frac{v^2}{(u^2+v^2)^{3/2}},$$

$$w_{uv}=u\left(-\tfrac{1}{2}\right)\left(u^2+v^2\right)^{-3/2}(2v)=-\frac{uv}{(u^2+v^2)^{3/2}},\quad w_{vu}=v\left(-\tfrac{1}{2}\right)\left(u^2+v^2\right)^{-3/2}(2u)=-\frac{uv}{(u^2+v^2)^{3/2}},$$

$$w_{vv}=\frac{1\cdot\sqrt{u^2+v^2}-v\cdot\frac{1}{2}(u^2+v^2)^{-1/2}(2v)}{\left(\sqrt{u^2+v^2}\right)^2}=\frac{\sqrt{u^2+v^2}-v^2/\sqrt{u^2+v^2}}{u^2+v^2}=\frac{u^2+v^2-v^2}{(u^2+v^2)^{3/2}}=\frac{u^2}{(u^2+v^2)^{3/2}}.$$

58. $v=e^{xe^y} \quad\Rightarrow\quad v_x=e^{xe^y}\cdot e^y=e^{y+xe^y},\ v_y=e^{xe^y}\cdot xe^y=xe^{y+xe^y}$. Then $v_{xx}=e^{y+xe^y}\cdot e^y=e^{2y+xe^y}$,

$v_{xy}=e^{y+xe^y}(1+xe^y),\quad v_{yx}=xe^{y+xe^y}(e^y)+e^{y+xe^y}(1)=e^{y+xe^y}(1+xe^y),$

$v_{yy}=xe^{y+xe^y}(1+xe^y)=e^{y+xe^y}(x+x^2e^y).$

67. $u=e^{r\theta}\sin\theta \quad\Rightarrow\quad \frac{\partial u}{\partial\theta}=e^{r\theta}\cos\theta+\sin\theta\cdot e^{r\theta}\,(r)=e^{r\theta}\,(\cos\theta+r\sin\theta),$

$$\frac{\partial^2 u}{\partial r\,\partial\theta}=e^{r\theta}\,(\sin\theta)+(\cos\theta+r\sin\theta)\,e^{r\theta}\,(\theta)=e^{r\theta}\,(\sin\theta+\theta\cos\theta+r\theta\sin\theta),$$

$$\frac{\partial^3 u}{\partial r^2\,\partial\theta}=e^{r\theta}\,(\theta\sin\theta)+(\sin\theta+\theta\cos\theta+r\theta\sin\theta)\cdot e^{r\theta}\;(\theta)=\theta e^{r\theta}\,(2\sin\theta+\theta\cos\theta+r\theta\sin\theta).$$

68. $z = u\sqrt{v-w} = u(v-w)^{1/2} \quad \Rightarrow \quad \dfrac{\partial z}{\partial w} = u\left[\frac{1}{2}(v-w)^{-1/2}(-1)\right] = -\frac{1}{2}u(v-w)^{-1/2}$,

$\dfrac{\partial^2 z}{\partial v\,\partial w} = -\frac{1}{2}u\left(-\frac{1}{2}(v-w)^{-3/2}(1)\right) = \frac{1}{4}u(v-w)^{-3/2}$, $\dfrac{\partial^3 z}{\partial u\,\partial v\,\partial w} = \frac{1}{4}(v-w)^{-3/2}$.