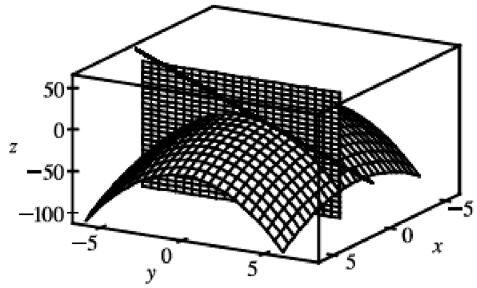
**94.** Setting $x = 1$, the equation of the parabola of intersection is $z = 6 - 1 - 1 - 2y^2 = 4 - 2y^2$. The slope of the tangent is $\partial z/\partial y = -4y$, so at $(1, 2, -4)$ the slope is $-8$. Parametric equations for the line are therefore $x = 1,\ y = 2 + t$, $z = -4 - 8t$.




**95.** By the geometry of partial derivatives, the slope of the tangent line is $f_x(1, 2)$. By implicit differentiation of $4x^2 + 2y^2 + z^2 = 16$, we get $8x + 2z\,(\partial z/\partial x) = 0 \quad \Rightarrow \quad \partial z/\partial x = -4x/z$, so when $x = 1$ and $z = 2$ we have $\partial z/\partial x = -2$. So the slope is $f_x(1, 2) = -2$. Thus the tangent line is given by $z - 2 = -2(x - 1),\ y = 2$. Taking the parameter to be $t = x - 1$, we can write parametric equations for this line: $x = 1 + t,\ y = 2,\ z = 2 - 2t$.

**3.** $z = f(x, y) = \sqrt{xy} \quad \Rightarrow \quad f_x(x, y) = \frac{1}{2}(xy)^{-1/2} \cdot y = \frac{1}{2}\sqrt{y/x}$, $f_y(x, y) = \frac{1}{2}(xy)^{-1/2} \cdot x = \frac{1}{2}\sqrt{x/y}$, so $f_x(1, 1) = \frac{1}{2}$ and $f_y(1, 1) = \frac{1}{2}$. Thus an equation of the tangent plane is $z - 1 = f_x(1, 1)(x - 1) + f_y(1, 1)(y - 1) \quad \Rightarrow$ $z - 1 = \frac{1}{2}(x - 1) + \frac{1}{2}(y - 1)$ or $x + y - 2z = 0$.

**5.** $z = f(x, y) = x\sin(x + y) \quad \Rightarrow \quad f_x(x, y) = x \cdot \cos(x + y) + \sin(x + y) \cdot 1 = x\cos(x + y) + \sin(x + y)$, $f_y(x, y) = x\cos(x + y)$, so $f_x(-1, 1) = (-1)\cos 0 + \sin 0 = -1$, $f_y(-1, 1) = (-1)\cos 0 = -1$ and an equation of the tangent plane is $z - 0 = (-1)(x + 1) + (-1)(y - 1)$ or $x + y + z = 0$.

**11.**
$f(x, y) = 1 + x\ln(xy - 5)$. The partial derivatives are $f_x(x, y) = x \cdot \dfrac{1}{xy - 5}\,(y) + \ln(xy - 5) \cdot 1 = \dfrac{xy}{xy - 5} + \ln(xy - 5)$ and $f_y(x, y) = x \cdot \dfrac{1}{xy - 5}\,(x) = \dfrac{x^2}{xy - 5}$, so $f_x(2, 3) = 6$ and $f_y(2, 3) = 4$. Both $f_x$ and $f_y$ are continuous functions for $xy > 5$, so by Theorem 8, $f$ is differentiable at $(2, 3)$. By Equation 3, the linearization of $f$ at $(2, 3)$ is given by $L(x, y) = f(2, 3) + f_x(2, 3)(x - 2) + f_y(2, 3)(y - 3) = 1 + 6(x - 2) + 4(y - 3) = 6x + 4y - 23$.

**14.** $f(x, y) = \sqrt{x + e^{4y}} = (x + e^{4y})^{1/2}$. The partial derivatives are $f_x(x, y) = \frac{1}{2}(x + e^{4y})^{-1/2}$ and $f_y(x, y) = \frac{1}{2}(x + e^{4y})^{-1/2}(4e^{4y}) = 2e^{4y}(x + e^{4y})^{-1/2}$, so $f_x(3, 0) = \frac{1}{2}(3 + e^0)^{-1/2} = \frac{1}{4}$ and $f_y(3, 0) = 2e^0(3 + e^0)^{-1/2} = 1$. Both $f_x$ and $f_y$ are continuous functions near $(3, 0)$, so $f$ is differentiable at $(3, 0)$ by Theorem 8. The linearization of $f$ at $(3, 0)$ is
$L(x, y) = f(3, 0) + f_x(3, 0)(x - 3) + f_y(3, 0)(y - 0) = 2 + \frac{1}{4}(x - 3) + 1(y - 0) = \frac{1}{4}x + y + \frac{5}{4}$.

**17.**
Let $f(x,y)=\dfrac{2x+3}{4y+1}$. Then $f_x(x,y)=\dfrac{2}{4y+1}$ and $f_y(x,y)=(2x+3)(-1)(4y+1)^{-2}(4)=\dfrac{-8x-12}{(4y+1)^2}$. Both $f_x$ and $f_y$ are continuous functions for $y\neq-\frac{1}{4}$, so by Theorem 8, $f$ is differentiable at $(0,0)$. We have $f_x(0,0)=2$, $f_y(0,0)=-12$ and the linear approximation of $f$ at $(0,0)$ is $f(x,y)\approx f(0,0)+f_x(0,0)(x-0)+f_y(0,0)(y-0)=3+2x-12y$.

**21.** $f(x,y,z)=\sqrt{x^2+y^2+z^2} \quad\Rightarrow\quad f_x(x,y,z)=\dfrac{x}{\sqrt{x^2+y^2+z^2}},\ f_y(x,y,z)=\dfrac{y}{\sqrt{x^2+y^2+z^2}}$, and $f_z(x,y,z)=\dfrac{z}{\sqrt{x^2+y^2+z^2}}$, so $f_x(3,2,6)=\frac{3}{7},\ f_y(3,2,6)=\frac{2}{7},\ f_z(3,2,6)=\frac{6}{7}$. Then the linear approximation of $f$ at $(3,2,6)$ is given by

$$\begin{aligned}f(x,y,z)&\approx f(3,2,6)+f_x(3,2,6)(x-3)+f_y(3,2,6)(y-2)+f_z(3,2,6)(z-6)\\&=7+\tfrac{3}{7}(x-3)+\tfrac{2}{7}(y-2)+\tfrac{6}{7}(z-6)=\tfrac{3}{7}x+\tfrac{2}{7}y+\tfrac{6}{7}z\end{aligned}$$

Thus $\sqrt{(3.02)^2+(1.97)^2+(5.99)^2}=f(3.02,1.97,5.99)\approx\frac{3}{7}(3.02)+\frac{2}{7}(1.97)+\frac{6}{7}(5.99)\approx 6.9914$.

**25.** $z=e^{-2x}\cos 2\pi t \quad\Rightarrow$
$dz=\dfrac{\partial z}{\partial x}dx+\dfrac{\partial z}{\partial t}dt=e^{-2x}(-2)\cos 2\pi t\,dx+e^{-2x}(-\sin 2\pi t)(2\pi)\,dt=-2e^{-2x}\cos 2\pi t\,dx-2\pi e^{-2x}\sin 2\pi t\,dt$

**31.** $dx=\Delta x=0.05$, $dy=\Delta y=0.1$, $z=5x^2+y^2$, $z_x=10x$, $z_y=2y$. Thus when $x=1$ and $y=2$,
$dz=z_x(1,2)\,dx+z_y(1,2)\,dy=(10)(0.05)+(4)(0.1)=0.9$ while
$\Delta z=f(1.05,2.1)-f(1,2)=5(1.05)^2+(2.1)^2-5-4=0.9225$.

**33.**
$dA=\dfrac{\partial A}{\partial x}\,dx+\dfrac{\partial A}{\partial y}\,dy=y\,dx+x\,dy$ and $|\Delta x|\le 0.1$, $|\Delta y|\le 0.1$. We use $dx=0.1$, $dy=0.1$ with $x=30$, $y=24$; then the maximum error in the area is about $dA=24(0.1)+30(0.1)=5.4\text{ cm}^2$.

**42.**
$\mathbf{r}_1(t)=\langle 2+3t,1-t^2,3-4t+t^2\rangle \quad\Rightarrow\quad \mathbf{r}_1'(t)=\langle 3,-2t,-4+2t\rangle,\quad \mathbf{r}_2(u)=\langle 1+u^2,2u^3-1,2u+1\rangle \quad\Rightarrow$
$\mathbf{r}_2'(u)=\langle 2u,6u^2,2\rangle$. Both curves pass through $P$ since $\mathbf{r}_1(0)=\mathbf{r}_2(1)=\langle 2,1,3\rangle$, so the tangent vectors $\mathbf{r}_1'(0)=\langle 3,0,-4\rangle$ and $\mathbf{r}_2'(1)=\langle 2,6,2\rangle$ are both parallel to the tangent plane to $S$ at $P$. A normal vector for the tangent plane is
$\mathbf{r}_1'(0)\times\mathbf{r}_2'(1)=\langle 3,0,-4\rangle\times\langle 2,6,2\rangle=\langle 24,-14,18\rangle$, so an equation of the tangent plane is
$24(x-2)-14(y-1)+18(z-3)=0$ or $12x-7y+9z=44$.

**4.** $z = \tan^{-1}(y/x)$, $x = e^t$, $y = 1 - e^{-t}$ $\Rightarrow$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = \frac{1}{1+(y/x)^2}(-yx^{-2}) \cdot e^t + \frac{1}{1+(y/x)^2}(1/x) \cdot (-e^{-t})(-1)$$

$$= -\frac{y}{x^2+y^2} \cdot e^t + \frac{1}{x+y^2/x} \cdot e^{-t} = \frac{xe^{-t} - ye^t}{x^2+y^2}$$

**5.** $w = xe^{y/z}$, $x = t^2$, $y = 1-t$, $z = 1+2t$ $\Rightarrow$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt} = e^{y/z} \cdot 2t + xe^{y/z}\left(\frac{1}{z}\right) \cdot (-1) + xe^{y/z}\left(-\frac{y}{z^2}\right) \cdot 2 = e^{y/z}\left(2t - \frac{x}{z} - \frac{2xy}{z^2}\right)$$

**13.** When $t = 3$, $x = g(3) = 2$ and $y = h(3) = 7$. By the Chain Rule (2),

$$\frac{dz}{dt} = \frac{\partial f}{\partial x}\frac{dx}{dt} + \frac{\partial f}{\partial y}\frac{dy}{dt} = f_x(2,7)g'(3) + f_y(2,7)\,h'(3) = (6)(5) + (-8)(-4) = 62.$$

**24.** $P = \sqrt{u^2+v^2+w^2} = (u^2+v^2+w^2)^{1/2}$, $u = xe^y$, $v = ye^x$, $w = e^{xy}$ $\Rightarrow$

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial u}\frac{\partial u}{\partial x} + \frac{\partial P}{\partial v}\frac{\partial v}{\partial x} + \frac{\partial P}{\partial w}\frac{\partial w}{\partial x}$$

$$= \tfrac{1}{2}(u^2+v^2+w^2)^{-1/2}(2u)(e^y) + \tfrac{1}{2}(u^2+v^2+w^2)^{-1/2}(2v)(ye^x) + \tfrac{1}{2}(u^2+v^2+w^2)^{-1/2}(2w)(ye^{xy})$$

$$= \frac{ue^y + vye^x + wye^{xy}}{\sqrt{u^2+v^2+w^2}},$$

$$\frac{\partial P}{\partial y} = \frac{\partial P}{\partial u}\frac{\partial u}{\partial y} + \frac{\partial P}{\partial v}\frac{\partial v}{\partial y} + \frac{\partial P}{\partial w}\frac{\partial w}{\partial y} = \frac{u}{\sqrt{u^2+v^2+w^2}}(xe^y) + \frac{v}{\sqrt{u^2+v^2+w^2}}(e^x) + \frac{w}{\sqrt{u^2+v^2+w^2}}(xe^{xy})$$

$$= \frac{uxe^y + ve^x + wxe^{xy}}{\sqrt{u^2+v^2+w^2}}.$$

When $x = 0$ and $y = 2$ we have $u = 0$, $v = 2$, and $w = 1$, so $\dfrac{\partial P}{\partial x} = \dfrac{0+4+2}{\sqrt{5}} = \dfrac{6}{\sqrt{5}}$ and $\dfrac{\partial P}{\partial y} = \dfrac{0+2+0}{\sqrt{5}} = \dfrac{2}{\sqrt{5}}$.

**34.** $yz + x\ln y = z^2$, so let $F(x,y,z) = yz + x\ln y - z^2 = 0$. Then $\dfrac{\partial z}{\partial x} = -\dfrac{F_x}{F_z} = -\dfrac{\ln y}{y-2z} = \dfrac{\ln y}{2z-y}$ and

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{z+(x/y)}{y-2z} = \frac{x+yz}{2yz-y^2}.$$

**38.** $V = \pi r^2 h/3$, so $\dfrac{dV}{dt} = \dfrac{\partial V}{\partial r}\dfrac{dr}{dt} + \dfrac{\partial V}{\partial h}\dfrac{dh}{dt} = \dfrac{2\pi rh}{3}\,1.8 + \dfrac{\pi r^2}{3}(-2.5) = 20{,}160\pi - 12{,}000\pi = 8160\pi \text{ in}^3/\text{s}$.

**39.** (a) $V = \ell w h$, so by the Chain Rule,

$$\frac{dV}{dt} = \frac{\partial V}{\partial \ell}\frac{d\ell}{dt} + \frac{\partial V}{\partial w}\frac{dw}{dt} + \frac{\partial V}{\partial h}\frac{dh}{dt} = wh\,\frac{d\ell}{dt} + \ell h\,\frac{dw}{dt} + \ell w\,\frac{dh}{dt} = 2\cdot 2\cdot 2 + 1\cdot 2\cdot 2 + 1\cdot 2\cdot(-3) = \mathbf{6}\ \mathrm{m}^3/\mathrm{s}.$$

(b) $S = 2(\ell w + \ell h + wh)$, so by the Chain Rule,

$$\begin{aligned}\frac{dS}{dt} &= \frac{\partial S}{\partial \ell}\frac{d\ell}{dt} + \frac{\partial S}{\partial w}\frac{dw}{dt} + \frac{\partial S}{\partial h}\frac{dh}{dt} = 2(w+h)\,\frac{d\ell}{dt} + 2(\ell+h)\,\frac{dw}{dt} + 2(\ell+w)\,\frac{dh}{dt}\\ &= 2(2+2)2 + 2(1+2)2 + 2(1+2)(-3) = \mathbf{10}\ \mathrm{m}^2/\mathrm{s}\end{aligned}$$

(c) $L^2 = \ell^2 + w^2 + h^2 \quad\Rightarrow\quad 2L\,\frac{dL}{dt} = 2\ell\,\frac{d\ell}{dt} + 2w\,\frac{dw}{dt} + 2h\,\frac{dh}{dt} = 2(1)(2) + 2(2)(2) + 2(2)(-3) = 0 \quad\Rightarrow$

$dL/dt = \mathbf{0}\ \mathrm{m/s}$.