Practice Midterm 2
Calculus III Section 8 - Fall 2013

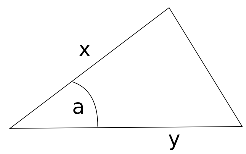
- The use of class notes, book, formulae sheet, calculator is **not permitted**.
- In order to get full credit, you **must**:
  a) get the **correct answer**, and
  b) **show all your work** and/or explain the reasoning that lead to that answer.

- Each solution must have a clearly labeled problem number and start at the top of a new page.
- Please make sure the solutions you hand in are **legible and lucid**. You may only use techniques we have developed in class.
- You have **one hour and fifteen minutes** to complete the exam.
- Do not forget to write your name and UNI in the space provided below and on the top of each page.

Enjoy the exam, and good luck!

FULL NAME (print): ______________________________ **UNI:** _______________

Exercise 1. [10 points] Fix a triangle T as in the picture defined by two edges of size x and y, and an angle a between them. Assume that the edge x is increasing at the rate of $3\,cm/sec$, y is decreasing at the rate of $2\,cm/sec$ and a is increasing at the rate of $0.5\,rad/sec$. Find the rate of change of the are of the triangle T when $x = 40\,cm$, $y = 50\,cm$ and $a = \pi/6\,rad$.



Exercise 2. [15 points] Evaluate the limit or show that it does not exist:

a) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{2x\sin y}{x^2+2y^2}$, b) $\displaystyle\lim_{(x,y)\to(0,0)} (x^2+y^2)\ln(x^2+y^2)$, c) $\displaystyle\lim_{(x,y)\to(1,0)} \frac{e^y(xy-y)}{\sqrt{x^2+y^2-2x+1}}$.

Exercise 3. [20 points] Consider the function $f\colon \mathbb{R}^2 \to \mathbb{R}$ defined by the formula

$$f(x,y) = \begin{cases} 0 & \text{if } y \le 0 \quad \text{or} \quad y \ge x^2, \\ 1 & \text{if } 0 < y < x^2. \end{cases}$$

a) Show that $f(x,y) \to 0$ as $(x,y) \to (0,0)$ along any line through $(0,0)$.

b) Despite part (a), show that f is discontinuous at $(0,0)$.

c) Show that f is discontinous along two entire curves and write these curves explicitly.

Exercise 4. [10 points] The ellipsoid $4x^2+3y^2+2z^2 = 16$ intersects the plane $y = 2$ in an ellipse. Find the parametric equations of the tangent line to this ellipse at the point $(1,2,0)$.

Exercise 5. [15 points] Consider the surface $yz + x\ln(y) - z^2 = 0$.

a) Find $\partial z/\partial x$ and $\partial z/\partial y$.

b) What is the domain of the function z? When is the function z differentiable?

Exercise 6. [30 points] True/False. Justify your answer with a proof if true, or a counterexample if false.

a) For two parameteric curves $\vec{r_1}(t)$ and $\vec{r_2}(t)$ we have $\dfrac{d(\vec{r_1}(t)\times\vec{r_2}(t))}{dt} = \vec{r_1}\,'(t)\times\vec{r_2}\,'(t)$.

b) The expression $1 + y/3$ gives a linear approximation to the function $\sqrt{y + cos^2 x}$ near the point $(0,0)$.

c) The domain of the function $f(x,y) = (sin^{-1}(x^2+y^2)\,ln(1+x^2), \dfrac{e^x}{x+y-1})$ is obtain by removing the line $x = 1-y$ from the disc of radius 1 centered at $(0,0)$.

d) The derivate $F_{\alpha,\alpha}$ of the function $F(\alpha,\beta) = \int_\beta^{\alpha^2} \sqrt{1+t^3}\,dt$ equals $\dfrac{2+5\alpha^3}{\sqrt{1+\alpha^3}}$.

e) The cross derivatives of the function $f(x,y) = x^3e^y - sin(y)ln(1+x^2)$ match.

f) The angle of intersection of the curves $\vec{r_1}(t) = \langle t, t^2, t^3\rangle$ and $\vec{r_2}(t) = \langle sin\,2t, sin\,4t, 2t\rangle$ is $\pi/2$.

1		2			3			4	5		6						**TOTAL**
a	b	a	b	c	a	b	c		a	b	a	b	c	d	e	f	