

Practice Midterm 1
Calculus III Section 8 - Fall 2013

- The use of class notes, book, formulae sheet, calculator is **not permitted**.
- In order to get full credit, you **must**:
 - a) get the **correct answer**, and
 - b) **show all your work** and/or explain the reasoning that lead to that answer.
- Each solution must have a clearly labeled problem number and start at the top of a new page.
- Please make sure the solutions you hand in are **legible and lucid**. You may only use techniques we have developed in class.
- You have **one hour and fifteen minutes** to complete the exam.
- Do not forget to write your name and UNI in the space provided below and on the top of each page.

Enjoy the exam, and good luck!

FULL NAME (print): _____ UNI: _____

Exercise 1. [15 points] Let $A = (1, 0, 1)$, $B = (3, 1, 0)$ and $C = (3, 2, 2)$ and $D = (-2, -2, 1)$ be four points in \mathbb{R}^3 .

- Find the volume of the parallelepiped formed by edges AB , AC and AD .
- Find the coordinates of the point E opposite to A in this parallelepiped.
- Find the angle $\angle EAB$.

Exercise 2. [16 points] Find the equation of the plane which containing the point $(6, 7, 0)$ and the line

$$L: \quad x - 1 = y - 2 = z/2.$$

Exercise 3. [15 points] Write the parametric equations describing the line

- passing through the point $(0, 1, 2)$,
- is parallel to the plane $x + y + z = 2$, and
- is perpendicular to line $x - 1 = 1 - y = 2z$.

Exercise 4. [24 points] True/False. Justify your answer with a proof if true, or a counterexample if false.

- $|\vec{v} \times \vec{u}|^2 = |\vec{u}|^2|\vec{v}|^2 - (\vec{u} \cdot \vec{v})^2$.
- $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$.
- $(\vec{u} - \vec{w}) \times (\vec{u} + \vec{w}) = 2(\vec{u} \times \vec{w})$.
- A plane Π in \mathbb{R}^3 can contain a pair of skew lines.
- The vectors $\vec{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}$, $\vec{v} = 3\mathbf{i} - \mathbf{j}$ and $\vec{w} = 5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$ are coplanar.
- The surface $x - 3y = z + 4y$ is a cylindrical surface.

Exercise 5. [25 points] Consider the equation $(a - 1)z = ax^2 + y^2$.

- Sketch the traces for $a = -1, 0, 1$.
- Sketch the surfaces for $a = -1, 0, 1$ and classify them.

Exercise 6. [15 points]

- Show that if $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are perpendicular, then the vectors \vec{u} and \vec{v} have the same length.
- Let \vec{w} be a nonzero vector. Prove that $\text{Orth}_{\vec{w}}(\vec{z}) = \vec{0}$ if and only if \vec{z} and \vec{w} are parallel.

1			2	3	4						5		6		TOTAL
a	b	c			a	b	c	d	e	f	a	b	a	b	