# Implicitization of Surfaces via Geometric Tropicalization

María Angélica Cueto

Department of Mathematics
University of California, Berkeley

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**Setting:** $f : \mathbb{A}^2 \to \mathbb{A}^3$ $(s,t) \mapsto (f_1(s,t), f_2(s,t), f_3(s,t))$ generically finite.

**GOAL:** Compute the defining equation of $Y = \overline{\mathrm{im} f}$.

**STRATEGY:** Compute $\mathcal{T}Y$ from $f$ ("Geometric Tropicalization").

$$\begin{tikzcd} X = \mathbb{A}^2 \setminus \bigcup_i (f_i = 0) \arrow[r, "f"] \arrow[d, hook] & \mathbb{G}_m^3 = \mathbb{T}^3 \\ \overline{X} \subset \mathbb{P}^2 \arrow[ur, "{(\frac{f_1^h}{u^{\deg f_1}}, \dots, \frac{f_3^h}{u^{\deg f_3}})}"'] & \end{tikzcd}$$

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PROBLEM: Compactification $\overline{X} \subset \mathbb{P}^2$ does NOT have nice properties in general (boundary divisors with combinatorial normal crossings). That is our four divisors $D_i := (f_i^h = 0)$ $i = 1, 2, 3$, and $D_\infty := (u = 0)$ might have triple intersections at points. (cfr. Example in Alicia's talk last week.)

SOLUTION: Blow-up at these points and use Thm [HKT] to get $\mathcal{T}Y$.

MY TASK: Make this resolution **explicit** and find **alternative ways** of building nice compactifications for $X$.