Practice Problems Final
Calculus I - Section 7 and 8
Fall 2011

Exercise 1. 1. *Sketch the graph of the function* $f(x) = \sqrt{x^2 + x} - x$. *Explain in detail all the calculations that led to your picture.*

2. *Sketch the graph of the function* $g(x) = 2(\sqrt{x^2 + x} - x) + 1$.

Exercise 2. *A particle moves along a horizontal line so that its coordinate at time* t *is* $x = \sqrt{b^2 + c^2t^2}$, $t \geq 0$, *where* b *and* c *are positive constants.*

1. *Find the velocity and acceleration functions.*

2. *Show that the particle always moves in the positive direction.*

Exercise 3. *The radius of a sphere is increasing at a rate of* $4\,mm/s$. *How fast is the volume increasing when the diameter is* $80\,mm$.

Exercise 4. *Sketch the region enclosed by the curves* $y = \sin x$, $y = \cos 2x$, $x = 0$ *and* $x = \pi/2$, *and find its area.*

Exercise 5. *Evaluate the following limits:*

1. $\lim\limits_{x \to 0} \frac{e^x - 1}{\tan x}$, 2. $\lim\limits_{x \to 0} \frac{\tan 4x}{x + sin 2x}$, 3. $\lim\limits_{x \to 1^-} \left(\frac{1}{x-1} + \frac{1}{x^2 - 3x + 2}\right)$.

Exercise 6. *If* f *is a continuous function such that*

$$\int_1^x f(t)\,dt = (x-1)e^{2x} + \int_1^x e^{-t} f(t)\,dt$$

for all x, *find an explicit formula for* $f(x)$.

Exercise 7. *Evaluate:*

1. $\int \sqrt{1 + x^2} x^5\,dx$, 2. $\int_{-1}^{3} 31 + x^2 x^5\,dx$, 3. $\int \tan x\,dx$. 4. $\int_0^{\pi/2} \tan x\,dx$.

Exercise 8. *Find the area of the largest rectangle that can be inscribed in the ellipse* $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Exercise 9. *Find* $\int_0^1 \frac{e^t + 1}{e^t + t}\,dt$ *and use this to evaluate* $\int_0^1 \frac{1-t}{e^t + t}\,dt$.

Exercise 10. *A cylindrical can without a top is made to contain* $V\,cm^3$ *of liquid. Find the dimensions that will minimize the cost of the metal to make the can.*

Exercise 11. *The acceleration of a car (in m/s^2) along a straight road is given by $a(t) = 3t - 5$ for $0 \leq t \leq 3$. The instant velocity of the car at time 0 is $8/3\,m/s$. Find the displacement and the distance traveled in 3 seconds.*

Exercise 12. *Let $f\colon [1, 11] \to \mathbb{R}$ be the piecewise defined function:*

$$f(x) = \begin{cases} 1 & \text{if } 1 \leq x < 2, \\ x - 1 & \text{if } 2 \leq x < 4, \\ -2x + 13 & \text{if } 4 \leq x < 5, \\ -x + 8 & \text{if } 5 \leq x < 9, \\ -1 & \text{if } 9 \leq x < 10, \\ x - 11 & \text{if } 10 \leq x \leq 11. \end{cases}$$

Let $g\colon [1, 11] \to \mathbb{R}$ be defined by $g(x) = \int\limits_1^x f(t)dt$.

1. *Is f continuous? Explain why.*
2. *Is f differentiable on $[1, 11]$? If not, find the points where f is not differentiable.*
3. *Draw the graph of f.*
4. *Evaluate g at $x = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10$ and 11.*
5. *On what interval is g increasing? On what interval is g decreasing? Describe the concavity of g. Find all critical points of g and all inflection points of g.*
6. *Sketch the graph of g.*

Exercise 13. *On what interval is the curve $g(x) = \int\limits_0^x \frac{t^2}{t^2+t+2}\,dt$ concave downwards?*

Exercise 14. *Find the volume common to two spheres, each with radius r if the center of each sphere lies on the surface of the other sphere.*

Exercise 15. *Find the tangent line to the curve $2(x^2 + y^2)^2 = 25(x^2 - y^2)$ at the point $(3, 1)$.*

Exercise 16. *For what values of x does the graph of $f(x) = e^x - 2x$ have a horizontal tangent?*

Exercise 17. *Let $f(x) = \ln(x - 1) - 1$*

1. *What is the domain and range of f?*
2. *What is the x-intercept of the graph of f?*
3. *Sketch the graph of f.*