

Practice Final – Math 1181H (Section 110) – Autumn 2017

Be sure to give complete explanations and show all your work. Let me know what you are thinking at every step.

1. (10 points) Determine if the following series are absolutely convergent, conditionally convergent, or divergent. Be sure to state which test you are using and verify the conditions.

$$(1) \sum_{n=2}^{\infty} (-1)^n \left( \frac{n+1}{2n} \right)^{\ln(n)}, \quad (2) \sum_{n=3}^{\infty} \frac{(-1)^n}{n \ln(n) (\ln(\ln(n)))^p} \quad \text{for } p > 0.$$

2. (10 points) (a) (4 points) Find the Taylor series expansion of  $e^{\sin(x)}$ .

(b) (3 points) Find the general term of the series  $x - \frac{x^3}{3^2} + \frac{x^5}{5^2} - \frac{x^7}{7^2} + \dots$ , determine its interval of convergence and compute its sum whenever it converges.

(c) (3 points) Compute the power series expansion of  $\frac{e^x}{2+x}$  around 0. What is the radius of convergence?

3. (10 points) Solve the equation  $y' = x - y$  with initial condition  $y(0) = 0$ . Can you recognize the solution as an elementary function?

4. (10 points) Compute the following limits:

$$(1) \lim_{x \rightarrow 0} \frac{\sqrt[3]{x+1} - (1+x/3)}{x^2}, \quad (2) \lim_{n \rightarrow \infty} \frac{1/n}{\sin(\pi/n)}, \quad (3) \lim_{x \rightarrow 0} (\cos(3x))^{1/x^3}, \quad (4) \lim_{x \rightarrow \pi} \frac{\ln(\cos(2x))}{(x-\pi)^2}.$$

5. (10 points) Consider the function

$$f(x) = \begin{cases} e^{-1/x^2} & \text{if } x \neq 0, \\ 0 & \text{otherwise} \end{cases}$$

(a) (4 points) Sketch the graph of  $f$  (find its maximum/minimum both local and absolute and its inflection points, compute  $\lim_{x \rightarrow \infty} f(x)$  and  $\lim_{x \rightarrow -\infty} f(x)$ )

(b) (3 points) Show that  $\lim_{x \rightarrow 0} x^{-n} f(x) = 0$

(c) (3 points) Show that for any  $n \geq 1$ ,  $f(x)$  has  $n$ th derivative at  $x \neq 0$  and from the formula conclude that  $f^{(n)}(0) = 0$  (using the definition of derivatives at  $x = 0$ ).

6. (10 points) Compute the following integrals:

$$(1) \int_0^2 \frac{\ln(x)}{\sqrt{x}} dx, \quad (3) \int_{1/\pi}^1 \frac{1}{x^2} \sin\left(\frac{1}{x}\right) dx, \quad (5) \int_0^5 \frac{1}{x^2(x^2+6x+5)} dx,$$

$$(2) \int_{-\infty}^{\infty} |x| e^{-x^2} dx, \quad (4) \int_1^{\infty} \cos(\ln(x)) dx, \quad (6) \int_{-1}^1 \frac{x}{\sqrt[3]{x+1}} dx.$$

7. (10 points) Two people start walking from the same point. One walks east at 3 mi/h and the other walks north at 2 mi/h. How fast is the distance between them changing after 15 minutes? (Hint: Draw a sketch of the movement)
8. (10 points) The top and bottom margins of a poster are each 6 cm and the side margins are each 4 cm. If the area of the printed material of the poster is fixed at  $348 \text{ cm}^2$ , find the dimensions of the poster with the smallest total area (printed material plus margins).
9. (10 points) Sketch the curves  $x = 8 - y^2$  and  $x = y^2 - 8$  and find the area enclosed by them.
10. (10 points) Find the volume and surface area of the cap of a sphere with radius  $r$  and height  $h$  as in the picture:

