Practice Midterm 2 – Math 1181H (Section 110) – Autumn 2017

Be sure to give complete explanations and show all your work. Let me know what you are thinking at every step.

1. (5 points) A ball is thrown vertically upward with an initial velocity of 78 ft/s from the roof of a building 400 ft high. Find the distance s from the ground up to the ball t seconds later. If the ball misses the building on the way down, how long does it take for it to hit the ground?

2. (15 points) Compute

$$(1) \lim_{x \to 0} \frac{tan(3x)}{4x}, \qquad (2) \lim_{n \to \infty} \left(1 + \frac{1}{2n^2}\right)^{2n}, \qquad (3) \frac{d}{dx} \int_0^{x^5} \frac{t\,dt}{\sqrt{1+t^2}}.$$

3. (10 points) Find the area under the arch of the curve $y = x^4 - 6x^2 + 9$ above the x-axis.

4. (10 points) Find $c > 0$ so that the area bounded by $y = x^2 - c$ and $y = c - x^2$ equals 9.

5. (10 points) Each plane perpendicular to the x-axis intersects a certain solid in a circular cross section whose diameter lies in the xy-plane and extends from $y = x^2$ to $y = 8 - x^2$. The solid lies between the points of intersection of these curves. Find its volume.

6. (10 points) Find the volume generated by revolving the area bounded by $x = y^2$ and $x = 4$ about: (1) the line $y = 2$, and (2) the line $x = -1$.

7. (10 points) Sketch the graph of $8a^2y^2 = x^2(a^2 - x^2)$ and find the surface area generated when this curve is revolved about the x-axis.

8. (10 points) A bag of sand is lifted at the constant rate of 3 ft/s for 10 seconds. At the beginning, the bag contains 100 lb of sand, but the sand leaks out at the rate of 4.5 lb/s. How much work is done in lifting this bag?

9. (10 points) Find the point on the graph of $f(x)$ at which the tangent line passes through the origin for (1) $f(x) = e^{ax}$, and (2) $f(x) = ln(x)$.

10. (10 points) Find the amplitude and frequency of the simple harmonic motion of a particle with trajectory $x(t) = 3\sin(2t) + 4\cos(2t)$. Find its maximal velocity.