

Quiz 1

NOTE: Answers without proper justification will receive NO credit

Problem 1. (3 points) Find the equation of the tangent line to the curve $y = (x^3 - x^2 + 1)^8$ at the point $(1, 1)$.

Write $f(x) = (x^3 - x^2 + 1)^8$, $f(1) = 1$. f is differentiable at $x=1$

The Tangent line has equation $y = f'(1)(x-1) + 1$

$$f'(x) = 8(x^3 - x^2 + 1)^7 \cdot (3x^2 - 2x) \quad \text{by the Chain Rule \& Power Rule}$$

$$\text{so } f'(1) = 8 \cdot 1 \cdot (3-2) = 8$$

$$\text{so } y = 8(x-1) + 1, \text{ that is } \boxed{y = 8x-7}$$

Problem 2. (2 points) Use the ε/δ method to show that $\lim_{x \rightarrow a} x^2 = a^2 = 0$.

want to find $\delta = \delta(\varepsilon) > 0$ so that if $0 < |x-a| < \delta$, then $|x^2 - a^2| < \varepsilon$

~~$|x^2 - a^2| = |x-a||x+a|$~~

If $|x| < \sqrt{\varepsilon}$, then $|x^2| < (\sqrt{\varepsilon})^2 = \varepsilon$ so we can

take $\delta = \sqrt{\varepsilon}$.

Note: for $\lim_{x \rightarrow a} x^2 = a^2$, we need to do more work:

$$|x^2 - a^2| = \underbrace{|x-a||x+a|}_{< \delta}$$

Since $\lim_{x \rightarrow a} x+a = 2a$, we know that we can find $\delta_1 > 0$ so

If $0 < |x-a| < \delta_1$, then $|x+a - 2a| < \varepsilon_1$ ($\delta_1 = \varepsilon_1$ works)

• If $a > 0$, take $\varepsilon_1 = a - 1a$, so $a - 1a < (x+a) < 2a + a = 3a$, so $|x+a| < 3a$.

• If $a < 0$, take $\varepsilon_1 = -a - 1a$, so $3a < x+a < a < 0$ so $|x+a| < 3|a|$

Then: $|x-a||x+a| < \delta \cdot 3|a| = \varepsilon$ $\Rightarrow \boxed{\delta = \min\{\frac{\varepsilon}{3|a|}, \varepsilon_1\} \text{ works!}}$