

## Quiz 4

NOTE: Answers without proper justification will receive NO credit

**Problem 1.** (3 points) Find the power series expansion of the solution  $y = y(x)$  to the differential equation  $x y' = y$ . What is its radius of convergence?

$$\text{Propose } y = \sum_{n=0}^{\infty} a_n x^n \Rightarrow y' = \sum_{n=1}^{\infty} n a_n x^{n-1}$$

$$x y' = \sum_{n=1}^{\infty} n a_n x^n \stackrel{?}{=} \sum_{n=0}^{\infty} a_n x^n \text{ gives } a_0 = a_0 \\ n a_n = a_n \Leftrightarrow (n-1)a_n = 0 \text{ for } n \geq 1$$

so for  $n=1$ :  $0 \cdot a_1 = 0 \therefore a_1 \text{ is free}$   
 for  $n > 1$   $(n-1) \neq 0$ , so  $a_n = 0$ .

Get  $y = a_0 + a_1 x + a_2 x^2 + \dots = a_0 + a_1 x + 0 x^2 + \dots = \boxed{a_1 x}$

Radius of convergence is  $\Rightarrow +\infty$  by definition

Note: Could have used separation of variables  $\frac{dy}{y} = \frac{dx}{x} \Rightarrow \ln y = \int \frac{dy}{y} = \int \frac{dx}{x}$   
 so  $y = e^{\ln x + C} = e^{\ln x} \cdot e^C = e^C x$ .

**Problem 2.** (2 points) Find the power series expansion of the function  $\frac{1}{(1+x)^2}$  and compute the interval of convergence of the series.

$$\int \frac{1}{(1+x)^2} dx = \frac{-1}{1+x} + C \stackrel{\uparrow}{=} -1 \left( \frac{1}{1-(x)} \right) + C = -1 \sum_{n=0}^{\infty} (-x)^n + C$$

If radius of conv.  $R$ , then  $R = \text{ROC of } \frac{1}{1-x}$ , but we know that is  $1$ . so  $\boxed{R=1}$

$$= \sum_{n=0}^{\infty} (-1)^{n+1} x^n + C \text{ for } |x| < 1.$$

$$\frac{1}{(1+x)^2} = \left( \sum_{n=0}^{\infty} (-1)^{n+1} x^n + C \right)' = \boxed{\sum_{n=1}^{\infty} n(-1)^{n+1} x^{n-1}} \text{ is the power series expansion}$$

End points:  $x = -1$ :  $\sum_{n=0}^{\infty} n(-1)^{n+1} (-1)^{n-1} = \sum_{n=0}^{\infty} n(-1)^{2n} = \sum_{n=0}^{\infty} n = +\infty$ , ROC = 1.  
 $x = 1$ :  $\sum_{n=0}^{\infty} n(-1)^{n+1}$  also diverges (the general term does not have  $\lim = 0$ ).

So  $\boxed{\text{Interval of convergence} = (-1, 1)}$