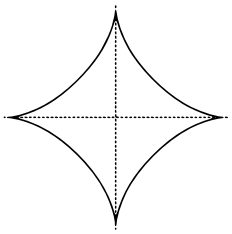
Practice Midterm 1 – Math 1181H (Section 110) – Autumn 2018

Be sure to give complete explanations and show all your work. Let me know what you are thinking at every step.

1. (15 points) Find the equation of the tangent line to the astroid curve $x^{2/3} + y^{2/3} = 4$ at the point $(-3\sqrt{3}, 1)$.



2. (20 points) The top of a ladder slides down a vertical wall at a rate of 0.15 m/s. At the moment when the bottom of the ladder is 3m from the wall, it slides away from the wall at a rate of 0.2 m/s. How long is the ladder?

3. (15 points) Find the inflection points, and the maximum and minimum values of the curve $y = x^3 - 3x + 2$ on the interval [0, 3]. Does this function have local maximum and minimum values on the same interval? Explain why.

4. (20 points) Compute the limits

$$\lim_{x\to\infty} 3\,x\tan(1/x) \qquad \text{and} \qquad \lim_{x\to\infty} (\sqrt{x+1} - \sqrt{x})$$

whenever these limits exist.

5. (15 points) The graph of a smooth differentiable function $f\colon [-1, 2] \to \mathbb{R}$ with $f(0) \neq 0$ has a point (x_0, y_0) that is closest to the origin. If this point satisfies $x_0 \neq -1, 2$, show that the line from the origin to (x_0, y_0) is a normal line to the graph.

6. (15 points) Consider the function $f\colon \mathbb{R} \to \mathbb{R}$ defined by:

$$f(x) = \begin{cases} x\,\sin(1/x) & \text{if } x \neq 0, \\ 0 & \text{if } x = 0. \end{cases}$$

   (a) (2 points) Show that the graph of f lies in between the lines $y = x$ and $y = -x$, and that the graph meets each of these two lines at infinitely many points.

   (b) (3 points) Show that $f(x)$ is continuous.

   (c) (2 points) Decide if the limits $\lim_{x\to\infty} f(x)$ and $\lim_{x\to-\infty} f(x)$ do exist.

   (d) (2 points) Find $f'(x)$ for $x \neq 0$

   (e) (2 points) Decide if the limits $\lim_{x\to\infty} f'(x)$ and $\lim_{x\to-\infty} f'(x)$ do exist.

   (f) (4 points) Show that $f'(0)$ does not exist.