

Practice Midterm 3 – Math 1181H (Section 110) – Autumn 2018

Be sure to give complete explanations and show all your work. Let me know what you are thinking at every step.

1. (20 points) Determine if the following series are absolutely convergent, conditionally convergent, or divergent. Be sure to state which test you are using and verify the conditions.

$$(1) \sum_{n=1}^{\infty} (-1)^n \frac{\ln(n)}{n^2},$$

$$(2) \sum_{n=2}^{\infty} \frac{(-1)^n}{n} e^n.$$

2. (10 points) Find the sum of the following series $\sum_{n=1}^{\infty} (-1)^n \frac{2^n}{5^{n/2}}$.

3. (10 points) Fix $a > 1$. Show that the series $\sum_{n=1}^{\infty} \frac{1}{a^{\ln n}}$ diverges if $a \leq e$ and converges if $a > e$.

4. (40 points) Compute the following integrals:

$$(1) \int_1^2 \frac{4x^2 + 2x + 4}{x^3 + 4x} dx$$

$$(2) \int_0^{\pi/4} \frac{1 + \cos(2x)}{\sin^2(2x)} dx,$$

$$(3) \int_1^3 \frac{dx}{\sqrt{x^2 + 9}}.$$

$$(4) \int_1^{\infty} \cos(\ln(x)) dx.$$

5. (20 points) Compute the following limits:

$$(1) \lim_{x \rightarrow 0^+} \frac{\ln(x)}{x^2 - x},$$

$$(2) \lim_{n \rightarrow \infty} \frac{\ln(n^{100})}{\sqrt[5]{n}},$$

$$(3) \lim_{x \rightarrow 0} (\cos(2x))^{1/x^2},$$

$$(4) \lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x}).$$