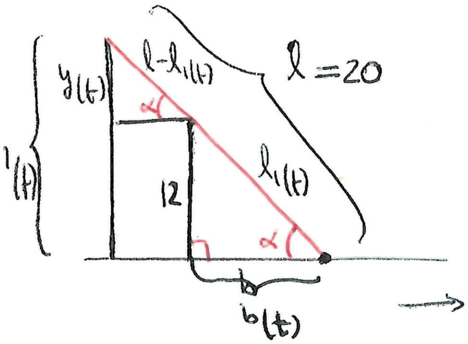
Problem 16 § 4.5



l = total length of ladder.

$h(t) = 12 + y(t)$

Relation between quantities

$$\boxed{b^2(t) + 12^2 = l_1^2(t)} \quad (*)$$

$$\frac{12}{l_1(t)} = \frac{y(t)}{(20 - l_1(t))} = \sin \alpha$$

So $\dfrac{12(20 - l_1(t))}{l_1(t)} = y(t)$

$$\rightsquigarrow \quad h(t) = 12 + y(t) = 12 + \frac{12(20 - l_1(t))}{l_1(t)} = \frac{240}{l_1(t)}$$

We want to use our original relation $(*)$ so $l_1(t) = \sqrt{12^2 + b^2(t)}$ gives

$$\boxed{h(t) = \frac{240}{\sqrt{144 + b^2(t)}}}$$

Then $$h'(t) = \frac{240}{(144 + b^2(t))^{3/2}} \left(\frac{-1}{2}\right) \cdot 2b(t)\, b'(t) = \frac{-240\, b(t)\, \boxed{b'(t)}}{\left(\sqrt{144 + b^2(t)}\right)^3}$$

$b'(t) = 5 \text{ ft/min}$

For (a) 5 ft project over means $l - l_1(t) = 20 - l_1(t) = 5$ so $l_1(t) = 15$

This gives $15^2 = 12^2 + b^2(t)$ so $b^2(t) = 81$ gives $\boxed{b_1(t) = 9}$

So $$h'(t) = \frac{-240 \cdot 9 \cdot 5}{15^3} = \boxed{\frac{-16}{5} \text{ ft/min}}$$

For (b): top of the ladder reaches top of the wall means $l_1(t) = l = 20$.

so $b(t) = \sqrt{20^2 - 12^2} = 16$

This gives $$h'(t) = \frac{-240 \cdot 16 \cdot 5}{20^3} = \boxed{\frac{-12}{5} \text{ ft/min}}$$