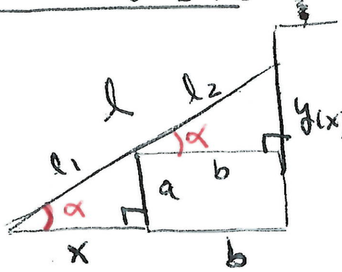
Problem 28 §4.3 Fireman problem. $\underline{A}$: $(\sqrt{b+1})\cdot(a+\sqrt{b})$



l = length of ladder $= l_1 + l_2$

- x determines everything so $l = l(x) = l_1(x) + l_2(x)$
- $x \geq 0$

Relation between variables? Use similarity of Δ.

angle α gives $\tan\alpha = \dfrac{a}{x}$ (from lower Δ)

$$\tan\alpha = \frac{y(x)}{b}$$

$$\frac{a}{x} = \frac{y(x)}{b}$$

$$\boxed{\frac{ab}{x} = y(x)} \quad (*)$$

We can write l_1 & l_2 via Pythagoras:

$$x^2 + a^2 = l_1^2(x) \qquad \& \qquad b^2 + y^2(x) = l_2^2(x)$$

Alternatively: $l_1(x) + l_2(x) = \sqrt{x^2+a^2} + \sqrt{b^2+y^2(x)}$

$$\underset{\text{use } (*)}{=} \sqrt{x^2+a^2} + \sqrt{b^2 + \frac{a^2b^2}{x^2}}$$

<u>Conclusion</u>: we need to <u>minimize</u> $l(x) = l_1(x) + l_2(x)$, for $x \geq 0$.

We know the min value exists since $\lim_{x\to\infty} l(x) = \infty$.

$$l(x) = \sqrt{x^2+a^2} + \sqrt{b^2\frac{(x^2+a^2)}{x^2}} = \sqrt{x^2+a^2}\left(1+\sqrt{\frac{b^2}{x^2}}\right)$$

$$= \sqrt{x^2+a^2}\left(1+\frac{b}{x}\right) \qquad (b, x > 0)$$

Since $l(x) \geq 0$, minimizing $l(x)$ or $l^2(x)$ will give the <u>same x</u>, so we choose $L(x) = l^2(x) = (x^2+a^2)\left(1+\frac{b}{x}\right)^2$

$$L'(x) = 2x\left(1+\frac{b}{x}\right)^2 + (x^2+a^2)\,2\left(1+\frac{b}{x}\right)\left(\frac{-b}{x^2}\right) = 2\left(1+\frac{b}{x}\right)\left(x \cancel{(1+b)} - \frac{b}{x^2}(x^2+a^2)\right)$$

$L'(x) = 0$ for $1 + \frac{b}{x} = 0$ or $x + b - b - \frac{ba^2}{x^2} = \frac{x^2 - ba^2}{x^2} = 0$

$\downarrow$ $b, x > 0$ so no meaningful solution!

$\hookrightarrow x^2 = ba^2$

$$\boxed{x = \sqrt{ba}}$$

If $x < \sqrt{ba}$ close to $\sqrt{ba}$: $L'(x) < 0$; if $x > \sqrt{ba}$ close to $\sqrt{ba}$: $L'(x) > 0$ so $\boxed{x = \sqrt{ba} \text{ is a } \underline{\text{minimum}}!}$

$\Rightarrow \boxed{l(\sqrt{ba}) = (\sqrt{b+1})(a+\sqrt{b})}$