

Quiz 1

NOTE: Answers without proper justification will receive NO credit

Problem 1. (3 points) Find the equation of the tangent line to the curve $y = (x^3 - x^2 + 1)^8$ at the point $(1, 1)$.

$$\text{Slope} = y'(1). \quad y'(x) = 8(x^3 - x^2 + 1)^7(3x^2 - 2x)$$

$$y'(1) = 8(1-1+1)^7(3-2) = 8$$

So the line has equation.

$$y = 8(x-1) + 1$$

Problem 2. (2 points) State and prove the additive law for derivatives by using the definition.

If f & g differentiable, then $f+g$ is differentiable &
 $(f+g)' = f' + g'$.

$$\begin{aligned} \text{Why? } (f+g)' &= \lim_{\Delta x \rightarrow 0} \frac{(f+g)(x+\Delta x) - (f+g)(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{f(x+\Delta x) + g(x+\Delta x) - f(x) - g(x)}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \left(\frac{f(x+\Delta x) - f(x)}{\Delta x} + \frac{g(x+\Delta x) - g(x)}{\Delta x} \right) \end{aligned}$$

$$\begin{aligned} \text{So by Additive Limit Law} \quad & \downarrow \Delta x \rightarrow 0 & \downarrow \Delta x \rightarrow 0 \\ & f'(x) & g'(x) \\ & = f'(x) + g'(x), \text{ as we wanted.} \end{aligned}$$