Math 1181H (§110) – Nov. 16, 2018 Full Name: ____________________

Quiz 3

NOTE: Answers without proper justification will receive NO credit.

Problem 1. *(2 points)* Compute $\lim\limits_{x\to\infty} \dfrac{2^x - 3^x}{x}$.

The limit has the form $\frac{-\infty}{\infty}$ (write $2^x - 3^x = \underbrace{3^x}_{\to\infty}\left(\overbrace{\left(\frac{2}{3}\right)^x - 1}^{\to -1}\right)$ as $x \to \infty$)

We use L'Hosp. $2^x = e^{(\ln 2)x}$ so $(2^x)' = \ln 2 \, e^{x\ln 2} = \ln 2 \cdot 2^x$

$3^x = e^{(\ln 3)x}$ so $(3^x)' = \ln 3 \, 3^x$.

$$\lim_{x\to\infty} \frac{2^x - 3^x}{x} = \lim_{x\to\infty} \frac{\ln 2\, 2^x - \ln 3 \cdot 3^x}{1} = \lim_{x\to\infty} \underbrace{3^x}_{\to\infty}\Big(\ln 2 \underbrace{\left(\tfrac{2}{3}\right)^x}_{\to 0} \underbrace{- \ln 3}_{\to -\ln 3}\Big)$$

$$= \boxed{-\infty}$$

Problem 2. *(3 points)* Decide if $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$ converges or not. Explain your choice.

- We use the limit comparison criteria. with $\frac{1}{n}$

$$\frac{\frac{1}{n}}{\frac{1}{n+\sqrt{n}}} = \frac{n+\sqrt{n}}{n} = 1 + \frac{1}{\sqrt{n}} \xrightarrow[n\to\infty]{} 1 \neq 0.$$

Since $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges, so does $\sum_{n\geq 1}^{\infty} \frac{1}{n+\sqrt{n}}$.

- Alternative $n + \sqrt{n} < 2n$ for $n \geq 2$

$$\frac{1}{n+\sqrt{n}} > \frac{1}{2n}$$

Since $\sum_{n=1}^{\infty} \frac{1}{2n}$ diverges, so does $\sum_{n=1}^{\infty} \frac{1}{n+\sqrt{n}}$.