Practice Midterm 1 – Math 2153 (Section 10)

1. Give examples of the following. Be as explicit as possible. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a continuous vector-valued function $\mathbf{r}(t)$ which is *not* differentiable at $t = 2$.
   (b) (2 points) Give an example of a vector-valued function which has constant curvature $\kappa \neq 0$.
   (c) (2 points) Give equations for two different planes in $\mathbf{R}^3$ which are parallel to the plane $z = 2x - 5y$.
   (d) (2 points) Give an example of a vector $\mathbf{u} \in \mathbf{R}^2$ for which $\mathbf{u} \cdot \left\langle \cos \frac{\pi}{5}, \sin \frac{\pi}{5} \right\rangle = 0$

2. Let $\mathbf{u} = \langle 3, 2, 2 \rangle$ and $\mathbf{v} = \langle 1, -4, 6 \rangle$.
   (a) (2 points) Compute $2\mathbf{u} - \mathbf{v}$.
   (b) (2 points) Decide if the angle between $\mathbf{u}$ and $\mathbf{v}$ is acute, right or obtuse.
   (c) (2 points) Compute $\mathrm{proj}_{\mathbf{u}}\mathbf{v}$.

3. (5 points) A ball is thrown from an intial height of $20\,\mathrm{m}$ above with an initial speed of $10\,\mathrm{m/s}$. and initial angle of $\frac{\pi}{3}$ radians with the ground. Give the position vector of the ball $\mathbf{r}(t)$ at time t.

4. (5 points) A weight of $5\,\mathrm{N}$ is tied to two strings, both fastened to the ceiling. The first string is tied to the weight at position $\langle 0, 0 \rangle$ and is fastened to the ceiling at position $\langle 3, 2 \rangle$ The second string is tied to the weight at position $\langle 0, 0 \rangle$ and is fastened to the ceiling at position $\langle -1, 2 \rangle$. Compute the force vectors which give the forces that the strings exhert on the weight.

5. (5 points) Compute the maximum and minimum curvature for the parametric curve
$$\mathbf{r}(t) = \langle 2\cos t, 3\sin t \rangle \qquad \text{for } 0 \leq t \leq 2\pi.$$

6. (5 points) Give an equation for the plane containing the line $\mathbf{r}(t) = \langle 3 + t, 2, 1 - t \rangle$ and the point $(2, -3, 1)$.

7. (5 points) Compute the normal and tangential components of acceleration (a_N and a_T) for a particle whose position at time t is given by the vector-valued function
$$\mathbf{r}(t) = \langle t^2, t + 2 \rangle.$$

8. (5 points) Decide if the following limits exist, justifying your answer accordingly:
$$(i) \quad \lim_{(x,y)\to(0,0)} \frac{6xy^2}{x^2 + 7y^4} \quad ; \quad (ii) \quad \lim_{(x,y)\to(0,0)} \frac{sin(x^2 + 4y^2)}{x^2 + 4y^2}.$$