Practice Midterm 2 – Math 2153 (Section 10)

1. Give explicit examples of the following properties. **You do NOT need to justify your answers.**
   (a) (2 points) Give an example of a function $f(x, y)$ continuous on $\mathbb{R}^2$ such that there are infinitely many points $(a, b) \in \mathbb{R}^2$ such that f has a local maximum at (a, b).
   (b) (2 points) Give an example of a continuous function $f = f(x, y)\colon \mathbb{R}^2 \to \mathbb{R}$ which is *not* differentiable at $(0, 0)$ and has a saddle point at $(0, 0)$.
   (c) (2 points) Give an example of a function $f(x, y)$ with domain $\mathbb{R}^2$ for which $f_x(0, 0)$ exists but $f_y(0, 0)$ does not exist.
   (d) (2 points) Sketch level curves for a function $f(x, y)$ with four local maximum and no local minima. Make sure to include enough level curves to illustrate these properties.

2. (5 points) A basin of circulating water is represented by the region $R = \{(x, y) : 0 \leq x, y \leq 1\}$. The velocity components of the water at position (x, y) in R are

$$\begin{cases} \text{the east-west velocity} & u(x, y) = x(1 + x)(-1 + 2y), \\ \text{the north-south velocity} & v(x, y) = y(y - 1)(-1 + 2x). \end{cases}$$

   These velocity components produce some flow patterns in the basin. Find the rates of change of the water speed in the x- and y-directions.

3. (a) (2 points) Compute $D_{\mathbf{u}}(3x^3y - y)$ where $\mathbf{u} = \left\langle \frac{-2}{\sqrt{5}}, \frac{-1}{\sqrt{5}} \right\rangle$.
   (b) (2 points) Find the unit vector $\mathbf{u} = \langle u_1, u_2, u_3 \rangle$ pointing in the direction of maximum increase for the function $f(x, y, z) = xyz^2$ at the point $(1, 1, 1)$.
   (c) (2 points) Find the tangent plane to the graph of the function $f(x, y) = \dfrac{\ln(xy)}{e^y}$ at the point $(e, 1, e^{-1})$.
   (d) (2 points) Compute $\dfrac{\partial z}{\partial x}$ at $(x, y) = (1, 0)$ if (x, y, z) satisfy the implicit equation

$$xy + yz^2 + xz = 7.$$

   (e) (2 points) Estimate the change in $z = xy^2 - x^2 + y$ when (x, y) changes from $(1, 2)$ to $(1.1, 1.9)$.
   (f) (2 points) Compute $\dfrac{\partial z}{\partial t}(\pi/4)$ where $z = x^2 - 3y^2 + 20$ and the point (x, y) lies in the curve $x = \cos t$, $y = \sin t$.

4. (8 points) Compute the local maximum, minimum values, saddle points and absolute maximum and minimum values for the function $f(x, y) = x - xy$ on the region in the plane bounded by the ellipse $y^2 + 9x^2 = 9$.

5. (5 points) Find the volume of the solid in the first octant bounded by the cylinder over the unit circle in the xy-plane and the planes $z = y$ and $z = 0$.

6. (a) (5 points) Find the area of the region inside both the cardioid $r = 1 - \cos\theta$ and the circle $r = 1$. (*Hint:* Draw a picture)
   (b) (5 points) Compute $\iint\limits_R 1\, dA$ where R is the region bounded by the curves $y = 3x^2$ and $y = 16 - x^2$.