1. Give explicit examples of the following properties. You do NOT need to justify your answers.

(a) (2 points) Give an example of a function \( f(x, y) \) continuous on \( \mathbb{R}^2 \) such that there are infinitely many points \((a, b) \in \mathbb{R}^2\) such that \( f \) has a local maximum at \((a, b)\).

(b) (2 points) Give an example of a continuous function \( f(x, y) : \mathbb{R}^2 \to \mathbb{R} \) which is not differentiable at \((0, 0)\) and has a saddle point at \((0, 0)\).

(c) (2 points) Give an example of a function \( f(x, y) \) with domain \( \mathbb{R}^2 \) for which \( f_x(0, 0) \) exists but \( f_y(0, 0) \) does not exist.

(d) (2 points) Sketch level curves for a function \( f(x, y) \) with four local maximum and no local minima. Make sure to include enough level curves to illustrate these properties.

2. (5 points) A basin of circulating water is represented by the region \( R = \{(x, y) : 0 \leq x, y \leq 1\} \). The velocity components of the water at position \((x, y)\) in \( R \) are

\[
\begin{align*}
\text{the east-west velocity} & \quad u(x, y) = x(1 + x)(-1 + 2y), \\
\text{the north-south velocity} & \quad v(x, y) = y(y - 1)(-1 + 2x).
\end{align*}
\]

These velocity components produce some flow patterns in the basin. Find the rates of change of the water speed in the \( x \)- and \( y \)-directions.

3. (a) (2 points) Compute \( D_u(3x^3y - y) \) where \( u = \left\langle \frac{-2}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right\rangle \).

(b) (2 points) Find the unit vector \( u = \langle u_1, u_2, u_3 \rangle \) pointing in the direction of maximum increase for the function \( f(x, y, z) = xyz^2 \) at the point \((1, 1, 1)\).

(c) (2 points) Find the tangent plane to the graph of the function \( f(x, y) = \frac{\ln(xy)}{e^y} \) at the point \((e, 1, e^{-1})\).

(d) (2 points) Compute \( \frac{\partial z}{\partial x} \) at \((x, y) = (1, 0)\) if \((x, y, z)\) satisfy the implicit equation

\[ xy + yz^2 + xz = 7. \]

(e) (2 points) Estimate the change in \( z = xy^2 - x^2 + y \) when \((x, y)\) changes from \((1, 2)\) to \((1.1, 1.9)\).

(f) (2 points) Compute \( \frac{\partial z}{\partial t}(\pi/4) \) where \( z = x^2 - 3y^2 + 20 \) and the point \((x, y)\) lies in the curve \( x = \cos t, y = \sin t \).

4. (8 points) Compute the local maximum, minimum values, saddle points and absolute maximum and minimum values for the function \( f(x, y) = x - xy \) on the region in the plane bounded by the ellipse \( y^2 + 9x^2 = 9 \).

5. (5 points) Find the volume of the solid in the first octant bounded by the cylinder over the unit circle in the \( xy \)-plane and the planes \( z = y \) and \( z = 0 \).

6. (a) (5 points) Find the area of the region inside both the cardioid \( r = 1 - \cos \theta \) and the circle \( r = 1 \). (Hint: Draw a picture)

(b) (5 points) Compute \( \iint_R 1 \, dA \) where \( R \) is the region bounded by the curves \( y = 3x^2 \) and \( y = 16 - x^2 \).