Quiz 1

1. (1 point) Decide if the following statement is true or false. You do NOT need to justify your answer.

   T  F  If \( \mathbf{u} = \langle u_1, u_2, u_3 \rangle \) is a nonzero vector then \( \text{proj}_\mathbf{u} \mathbf{u} = \mathbf{u} \).

\[
\frac{\mathbf{u} \cdot \mathbf{u}}{\|\mathbf{u}\|^2} \cdot \mathbf{u} = \frac{1}{\|\mathbf{u}\|^2} \mathbf{u} \| \mathbf{u} \| = \mathbf{u} \quad \text{True}
\]

2. (1 point) Give a concrete example of a vector \( \mathbf{u} = \langle u_1, u_2 \rangle \) satisfying \( u_1 u_2 = 4 \). You do NOT need to simplify your answer.

\[
\mathbf{u} \cdot \mathbf{u} = \| \mathbf{u} \|^2 \text{ must equal } 4 \quad \text{so } \| \mathbf{u} \| = 2
\]

\[
\text{so } \mathbf{u} = \langle a, b \rangle \text{ satisfies } a^2 + b^2 = 4, \text{ pick } a = 0, b = 2
\]

\[
\mathbf{u} = \langle 0, 2 \rangle \text{ is an example}
\]

3. (3 points) A weight is suspended from the ceiling by two strings as pictured below. If each string can support a maximum tension of 10 lbs., what is the maximum weight \( W \) that this configuration can support before at least one string breaks?

\[
\text{Weight vector } \mathbf{W} = W \langle 0, -1 \rangle \quad \text{Need to find}
\]

\[
\mathbf{s}_1 + \mathbf{s}_2 = -\mathbf{W} \langle 0, -1 \rangle
\]

\[
\mathbf{W} = 10 \langle \cos 45^\circ, -\sin 45^\circ \rangle = 10 \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle
\]

\[
\implies \mathbf{s}_1 = \langle -\frac{\sqrt{2}}{2} s_1, \frac{\sqrt{2}}{2} s_1 \rangle
\]

\[
\text{Similarly: } \quad \mathbf{s}_2 = 10 \langle -\cos 45^\circ, -\sin 45^\circ \rangle = -10 \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle
\]

\[
\implies \mathbf{s}_2 = \langle -\frac{\sqrt{2}}{2} s_2, \frac{\sqrt{2}}{2} s_2 \rangle
\]

\[
\text{So } \mathbf{W} = 10 \langle 0, -1 \rangle = \mathbf{s}_1 + \mathbf{s}_2 = \langle 0, -10 \rangle \implies \mathbf{W} = 10 \langle \frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2} \rangle
\]