Math 2153 – February 4, 2016 Full Name: ______________________________

Quiz 2

1. *(1 point)* Give a concrete example of a vector-valued function $\mathbf{r}(t) = \langle x(t), y(t)\rangle$ for which $\mathbf{r}(t)$ is defined but not continuous for some value of t. **You do NOT need to justify your answer.**

Easiest example: partition function with 2 different constant values

$$\vec r(t) = \begin{cases} \langle 0,0\rangle & t \neq 0 \\ \langle 1,1\rangle & t = 0. \end{cases}$$

define on $t=0$ but not continuous at $t=0$.

2. *(2 points)* Consider the lines

$$\mathbf{r}(t) = \langle 2+t, 3-4t, -2\rangle$$

and

$$\mathbf{q}(s) = \langle 7+2s, 1-2s, 1+s\rangle$$

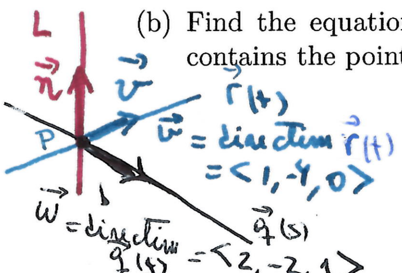
(a) Show that the point $P = (1, 7, -2)$ belongs to both lines.

$\vec r(t) = \langle 1,7,-2\rangle \iff 2+t=1$, $\boxed{t=-1}$, $3-4t=7$, $t=-1$ ✓ & $-2=-2$ ✓

$q(s) = \langle 1,7,-2\rangle \iff 7+2s=1$, $\boxed{s=-3}$, $1-2s=7$, $s=-3$ ✓ & $1+s=-2$, $s=-3$ ✓

so $\boxed{\vec r(-1) = \vec s_{(3)} = P}$

(b) Find the equation of the line that is perpendicular to both lines $\mathbf{r}(t)$ and $\mathbf{q}(s)$ and contains the point P.

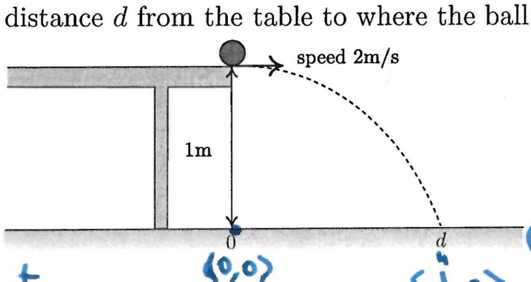


Line L passes through P & has direction $\vec n \perp \vec v$ & $\vec n \perp \vec w$

$$\vec n = \vec v \times \vec w = \begin{vmatrix} \vec i & \vec j & \vec k \\ 1 & -4 & 0 \\ 2 & -2 & 1 \end{vmatrix} = \vec i(-4) - \vec j(1) + \vec k(-2+8) = \langle -4,-1,6\rangle$$

$\boxed{L: \vec \ell(t) = \langle 1,7,-2\rangle + t\langle -4,-1,6\rangle}$ for t in $\mathbb{R}$

3. *(2 points)* A ball rolls of the edge of a table 1 m high with the speed of 2 m/s. Find the distance d from the table to where the ball lands.



$\vec a(t) = \langle 0,-g\rangle$ $\quad g = 9.8$

Initial conditions: $\vec r(0) = \langle 0,1\rangle$

$\vec v(0) = 2\langle 1,0\rangle = \langle 2,0\rangle$

(1) Integrate to get $\vec v(t)$

$\vec v(t) = \int_0^t \langle 0,-g\rangle\, ds = \langle 0,-gt\rangle + \vec C$ & use initial value $\vec v(0)$ to find $\vec C$: $\langle 2,0\rangle = \vec v(0) = \vec 0 + \vec C = \vec C$ so $\vec v(t) = \langle 2,-gt\rangle$

(2) Integrate $\vec v(t)$ to get $\vec r(t)$: $\vec r(t) = \int \langle 2,-gt\rangle\, dt = \langle 2t, -\frac{gt^2}{2}\rangle + \vec C_2$

Use $\vec r(0) = \langle 0,1\rangle = \vec 0 + \vec C_2 = \vec C_2$ to get $\vec r(t) = \langle 2t, -\frac{gt^2}{2} + 1\rangle$

(3) The ball hits the ground at time t_0 satisfying $-\frac{gt_0^2}{2} + 1 = 0$ so $t_0 = \sqrt{\frac{2}{g}}$

The distance d is the x-component of $\vec r(t_0)$ so $d = 2\sqrt{\frac{2}{g}} = \boxed{2/\sqrt{4.9}\text{ m}}$