Math 2153 – March 10, 2016 Full Name: ____________________

Quiz 3

1. *(1 point)* Find the tangent plane to the graph of $f(x,y) = e^{x^2+y^2} - e^2$ at the point $(1,1,0)$.

Normal direction $= \langle -f_x, -f_y, 1\rangle_{(1,1,0)} = \langle -2e^2, -2e^2, 1\rangle$

$\nabla f(x,y) = \langle e^{x^2+y^2}(2x), e^{x^2+y^2}(2y)\rangle$

$\Rightarrow$ Plane: $\langle -2e^2, -2e^2, 1\rangle \bullet \langle x, y, z\rangle = \langle -2e^2, -2e^2, 1\rangle \bullet \langle 1,1,0\rangle = -4e^2$

$\boxed{-2e^2x - 2e^2y + z = -4e^2}$

2. *(1 point)* Compute the double integral $\iint_R y\cos(xy)\, dA$ where $R = \{(x,y) : 0 \le x \le 1, 0 \le y \le \frac{\pi}{2}\}$

R is a rectangular region and $f(x,y) = y\cos xy$ is cont. on R, so we can use Fubini

$\iint_R f(x,y)\,dA = \int_0^{\pi/2}\Big(\underbrace{\int_0^1 y\cos xy\, dx}_{=\frac{\partial}{\partial x}(\sin(xy))}\Big)dy = \int_0^{\pi/2} \sin xy \Big|_{x=0}^{x=1} dy = \int_0^{\pi/2} (\sin y - 0)\,dy$

$= -\cos y \Big|_0^{\pi/2} = -(0-1) = \boxed{1}$

3. *(3 points)* Find the local min/max values, saddle points and absolute min and max values of the function $f(x,y) = xy$ on the region $D = \{(x,y) : x^2 + y^2 \le 8\}$

(a) Compute the critical pts: $\nabla f = 0$ (because f is differentiable up to any order)

$\nabla f = \langle y, x\rangle = \langle 0,0\rangle$ so only 1 crit pt. Use the Second Derivative Test:

$\left.\begin{array}{l} f_{xx} = f_{yy} = 0 \\ f_{xy} = f_{yx} = 1 \end{array}\right\}$ $D(x,y) = f_{xx}f_{yy} - f_{xy}^2 = -1 < 0 \Rightarrow (0,0)$ is a saddle pt.

Local Max & min $\rightarrow$ only on boundary pts.

(b) To find extremal values on the boundary, we use Lagrange multipliers.

Boundary: $g(x,y) = x^2+y^2-8$

$\nabla g = \langle 2x, 2y\rangle$

$\begin{cases} \nabla f = \lambda \nabla g \leftrightarrow y = 2\lambda x \ (*) \ \& \ x = 2\lambda y \ (1) \\ x^2+y^2 = 8 \end{cases}$

If $x = 0 \Rightarrow y = 0$ by $(*)$ but $0^2+0^2 \ne 8$ so $x \ne 0 \Rightarrow \boxed{\lambda = \frac{y}{2x}}$ from $(*)$

Substitute in (1) $x = 2\lambda y = \frac{2y^2}{2x} \Rightarrow 2x^2 = 2y^2$, $x^2 = y^2$ & $x^2+y^2 = 2x^2 = 8$ $\boxed{x = \pm 2}$

$\Rightarrow$ 4 pts: $(2,2)$, $(2,-2)$, $(-2,2)$, $(-2,-2)$.

$f(2,2) = f(-2,-2) = 4$. $f(-2,2) = f(2,-2) = -4$

$\downarrow$ ABS MAX $\downarrow$ ABS MIN